Eur. Phys. J. D (2017) 71: 229
DOI: 10.1140/epjd/e2017-80109-7

THE EUROPEAN PHYSICAL JOURNAL D

Regular Article

# Atoms in the counter-propagating frequency-modulated waves: splitting, cooling, confinement

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Received 17 February 2017 / Received in final form 5 May 2017
Published online 5 September 2017 – © EDP Sciences, Società Italiana di Fisica, Springer-Verlag 2017

**Abstract.** We show that the counter-propagating frequency-modulated (FM) waves of the same intensity can split an orthogonal atomic beam into two beams. We calculate the temperature of the atomic ensemble for the case when the atoms are grouped around zero velocity in the direction of the waves propagation. The high-intensity laser radiation with a properly chosen carrier frequency can form a one-dimensional trap for atoms. We carry out the numerical simulation of the atomic motion (two-level model of the atom-field interaction) using parameters appropriate for sodium atoms and show that sub-Doppler cooling can be reached. We suppose that such a cooling is partly based on the cooling without spontaneous emission in polychromatic waves [H. Metcalf, Phys. Rev. A **77**, 061401 (2008)]. We calculate the state of the atom in the field by the Monte Carlo wave-function method and describe its mechanical motion by the classical mechanics.

## 1 Introduction

The methods of laser control of atomic motion have been developing rapidly for more than four decades starting in 1970 by Ashkin in his paper [1], where he showed that significant deviation of the atomic beam by laser radiation is possible in the laboratory. Various ways to control the mechanical motion of atoms and their cooling are described, for an example, in reviews [2–5] and textbooks [6,7]. The laser radiation used in experiments may be monochromatic or polychromatic (bichromatic, pulsed, amplitude- or frequency-modulated). For example, in physical laboratories cold atoms are typically confined by magneto-optical trap with monochromatic laser radiation and magnetic field [7].

Recently it was shown that the sequences of counter-propagating trains of light pulses can form a trap for atoms [8–12]. Moreover, there is no need for additional fields to cool atoms down to the Doppler cooling limit [13–15]. Two collinear standing waves of the equal intensity and different frequencies are also expected to form a trap for atoms, as was shown in [16,17]. These waves can also be treated as a superposition of the counter-propagating amplitude-modulated bichromatic waves. In this case, as in the case of counter-propagating light pulses, simultaneous confinement and cooling of atoms by the same field are possible.

Light pressure force and behavior of atoms in the field of frequency-modulated (FM) waves was investigated earlier in the papers [18–27]. Wide spectrum of FM radiation allows to effective slow [18–21] and collimate [27] atomic beams by FM waves.

A new and unexpected direction of research is a laser cooling without the participation of the spontaneous emission, which was predicted in [28] and experimentally confirmed in [29,30] for an example of the interaction of He atoms with the counter-propagating bichromatic waves.

The aim of this paper is to consider the various aspects of the interaction of atoms with the counter-propagating FM waves. In particular, we found that such light waves can form a trap for atoms. Besides that, we predict laser cooling of atoms by the counter-propagating FM waves below Doppler limit for the specific parameters of the atom-field interaction. The calculations were carried out for the two-level model of the atom-field interaction using parameters appropriate for sodium atoms. The sub-Doppler cooling of atoms in the field of FM-waves is caused, at least in part, by laser cooling of the atomic ensemble without the participation of the spontaneous emission.

We also study how the momentum diffusion affects the motion of atoms and show that sometimes an atomic beam, orthogonal to the direction of propagation of FM waves, is divided into 2–3 beams contrary to the prediction, based on the velocity dependence of the light pressure force, of grouping of atoms around the zero velocity. The atomic ensemble temperature corresponding to the degree of freedom along the direction of wave propagation is calculated analytically (weak field) and numerically.

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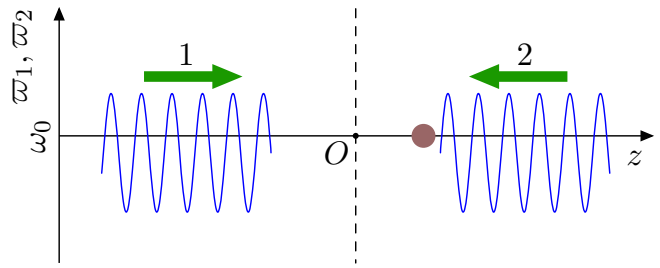


**Fig. 1.** The scheme of the interaction of an atom with the counter-propagating FM waves. The atom (indicated by circle) near the point $O$ is subjected to the field of the counter-propagating waves 1, 2 obtained from the same radiation source. The optical paths of the counter-propagating waves to the center of the trap (the point $O$ in the figure) are equal. The instant frequencies of the oncoming waves are equal to $\varpi_1$, $\varpi_2$; $\omega_0$ is the transition frequency of the atom.

We use the quasiclassical approximation for numerical simulation of the statistical characteristics of an atomic ensemble. The mechanical motion of atoms is described by classical mechanics taking into account the stochastic changes of the atomic momentum due to spontaneous emission. The state of the atom is calculated by Monte Carlo wave-function method [31].

The paper is structured as follows. In the next section, the scheme of the atom-field interaction is described, the basic equations used in the paper are presented in the third section, the temperature of the atomic ensemble in the field of low-intensity laser radiation is calculated in the fourth section. The fifth section describes the Monte Carlo wave function method, the sixth section gives the procedure for numerical calculation. The results obtained in the paper are discussed in seventh section. Finally, there is a brief summary.

## 2 Scheme of the atom-field interaction

We assume that an atom interacts with the counter-propagating FM waves 1, 2 obtained from the same radiation source (Fig. 1). The frequencies of the waves are modulated by the sinusoidal law. Keeping in mind the possible formation of a trap for atoms by these waves, we consider the motion of atoms near the point $O$ in which the counter-propagating waves come with the same phase. As a result, the force exerted on the atom in the point $O$ is equal to zero. Below we show that the counter-propagating FM waves can confine atoms in a small region near point $O$ (the center of the trap), provided that the parameters of the atom-field interaction are properly chosen.

## 3 Main equations

Consider an atom in the field of two counter-propagating FM waves produced by the same source. As a result, each of them repeats the other with some time delay. The origin of the coordinates is situated in the point $O$ (see Fig. 1), where this delay is equal to zero. The strength of the electric field exerted on the atom in the point $z$ equals

$$\boldsymbol{E} = \frac{1}{2}\boldsymbol{e}E_0\left[\exp\left(i\omega t - ikz + i\varphi_1\right) + \exp\left(i\omega t + ikz + i\varphi_2\right)\right] + \text{c.c.} \quad (1)$$

Here $\omega$ is the carrier frequency of the counter-propagating waves, $k = \omega/c$ is the wave vector, $\boldsymbol{e}$ is the unit vector of the polarization, $E_0$ is the amplitude of the electric field,

$$\varphi_1 = \beta\sin\left[\Omega_m\left(t - z/c\right)\right], \quad (2)$$

$$\varphi_2 = \beta\sin\left[\Omega_m\left(t + z/c\right)\right], \quad (3)$$

$\Omega_m$ is the modulation frequency, $\beta$ is the modulation index. The instant frequencies of the counter-propagating waves are

$$\varpi_1 = \omega + \dot{\varphi_1} = \omega + \beta\Omega_m\cos\left[\Omega_m\left(t - z/c\right)\right], \quad (4)$$

$$\varpi_2 = \omega + \dot{\varphi_2} = \omega + \beta\Omega_m\cos\left[\Omega_m\left(t + z/c\right)\right]. \quad (5)$$

These instant frequencies are the same, in particular, in $z = 0$ (the point $O$, i.e. the center of the trap) and points with $z$-coordinate equal to multiple of $2\pi c/\Omega_m$.

We use the two-level model of the atom. The energy difference between ground $|1\rangle$ and excited $|2\rangle$ states is $\hbar\omega_0$. The light pressure force on the atom along the $z$ axis is given by the formula [6,7]

$$F = (\varrho_{12}\boldsymbol{d}_{21} + \varrho_{21}\boldsymbol{d}_{12})\frac{\partial \boldsymbol{E}}{\partial z}, \quad (6)$$

where $\boldsymbol{d}_{12}$ and $\boldsymbol{d}_{21}$ are the matrix elements of the dipole moment, $\varrho_{12}$ and $\varrho_{21}$ are the nondiagonal elements of the density matrix $\varrho$. The atom moves according to Newton's law

$$\dot{v} = F/m, \quad (7)$$

where $m$ is the mass of the atom, $v$ is its velocity and $F$ is described by equation (6).

We assume that the condition [6,32]

$$\frac{\hbar^2k^2}{2m} \ll \hbar\gamma, \quad (8)$$

of the semi-classical treatment of the atomic motion is valid. This criterion means that that the light-pressure force is formed faster than the change of the atomic velocity will have a significant impact on its value (the heavy atom approximation). Here $\gamma$ is the rate constant of spontaneous emission.

After averaging over the period of the field's oscillations $2\pi/\omega_0$ the expression for the light pressure force equation (6) becomes

$$F = \hbar k\,\mathrm{Im}\left[\varrho_{12}e^{-i\omega_0 t + i\delta t}\left(\Omega_1^* e^{ikz} - \Omega_2^* e^{-ikz}\right)\right]. \quad (9)$$

Here $\Omega_1 = \Omega_R e^{i\varphi_1}$, $\Omega_2 = \Omega_R e^{i\varphi_2}$, $\Omega_R = -\boldsymbol{d}_{12}\boldsymbol{e}E_0/\hbar$, $\delta = \omega_0 - \omega$. The instantaneous Rabi frequencies $\Omega_1$, $\Omega_2$ can be written in the form of the Fourier series

$$\Omega_1(t - z/c) = \Omega_R\sum_{n=-\infty}^{\infty} J_n(\beta)e^{in\Omega(t - z/c)}, \quad (10)$$

$$\Omega_2(t + z/c) = \Omega_R\sum_{n=-\infty}^{\infty} J_n(\beta)e^{in\Omega(t + z/c)}, \quad (11)$$

where $J_n(\beta)$ are Bessel functions.

The density matrix we calculate by two methods. We found it from the density matrix equation

$$i\hbar\frac{\partial}{\partial t}\varrho_{jk}=\sum_l(H_{jl}\varrho_{lk}-\varrho_{jl}H_{lk})+i\hbar\sum_{l,m}\Gamma_{jk,lm}\varrho_{lm} \quad (12)$$

with Hamiltonian $H$

$$H=\hbar\omega_0|2\rangle\langle 2|-\boldsymbol{d}_{12}|1\rangle\langle 2|\boldsymbol{E}(t)-\boldsymbol{d}_{21}|2\rangle\langle 1|\boldsymbol{E}(t) \quad (13)$$

and matrix $\Gamma$ with nonzero elements

$$\begin{aligned}&\Gamma_{12,12}=\Gamma_{21,21}=-\gamma/2,\\&\Gamma_{11,22}=-\Gamma_{22,22}=\gamma\end{aligned} \quad (14)$$

in our investigation of the atomic motion in the weak field. These approach gives us the temperature of the atomic ensemble. We use the probability amplitudes of the wave function found by Monte Carlo wave-function method [31] for constructing the density matrix in our numerical simulation of the atomic ensemble evolution. In this case we solve the Schrödinger equation

$$i\hbar\frac{d}{dt}|\psi\rangle=H|\psi\rangle \quad (15)$$

with non-Hermitian Hamiltonian

$$H_{MC}=H-\frac{i\hbar\gamma}{2}|2\rangle\langle 2|, \quad (16)$$

and find the atomic state vector

$$|\psi\rangle=c_1|1\rangle+c_2e^{-i\omega_0t}|2\rangle, \quad (17)$$

taking into account the possible quantum jumps (see Sect. 5). We express the elements of the density matrix in terms of $c_1$ and $c_2$ by

$$\varrho_{12}=c_1c_2^*e^{i\omega_0t},\quad \varrho_{21}=c_2c_1^*e^{-i\omega_0t} \quad (18)$$

and then use them in the expression (6) for the light pressure force.

## 4 Atoms in a weak field

In this section, we analyze the behavior of the atomic ensemble in the weak FM waves, $\Omega_R\ll\gamma$. First of all, we estimate the temperature of the atomic ensemble. For this purpose, we need to find the average decelerating force exerted on the atom moving with a small velocity and the average number of photons which are emitted spontaneously by the atom per unit time [33]. The average number of the spontaneously emitted photons is determined by the population of the excited state. In order to find these quantities, we analyze the population of the excited state and light pressure force exerted on the atom in the weak field using the equations for the density matrix. At the end of the section, we show that under certain condition the light pressure force is directed towards the point with coordinate $z=0$, that is a prerequisite for the formation of a trap for atoms by the counter-propagating FM waves.

To calculate the light pressure force by equation (9), we, first of all, solve the equation for density matrix (12). After substitution of equations (13), (14) and (1) in equation (12) and applying rotating wave approximation (RWA) [34] we get

$$\begin{aligned}\frac{\partial}{\partial t}\varrho_{11}&=\frac{i}{2}\sigma_{12}e^{i\delta t}\left(\Omega_1^*e^{ikz}+\Omega_2^*e^{-ikz}\right)\\&\quad-\frac{i}{2}\sigma_{21}e^{-i\delta t}\left(\Omega_1e^{-ikz}+\Omega_2e^{ikz}\right)+\gamma\varrho_{22},\\\frac{\partial}{\partial t}\sigma_{12}&=\frac{i}{2}(\varrho_{11}-\varrho_{22})e^{-i\delta t}\left(\Omega_1e^{-ikz}+\Omega_2e^{ikz}\right)-\frac{\gamma}{2}\sigma_{12},\\\sigma_{21}&=\sigma_{12}^*,\quad \varrho_{11}+\varrho_{22}=1.\end{aligned} \quad (19)$$

Here

$$\sigma_{12}=\varrho_{12}e^{-i\omega_0t},\quad \sigma_{21}=\varrho_{21}e^{i\omega_0t}. \quad (20)$$

The solution to equation (19) becomes quasistationary when $t\gg\gamma^{-1}$. To calculate the light pressure force (9) up to the fourth order in the field strength, as it follows from equation (19), we must calculate $\sigma_{12}$, $\sigma_{21}$ up to the third order. In addition, we are interested in the population of the excited state $\varrho_{22}$ up to the second order. Let us introduce the parameter $\varepsilon$ to denote the order of smallness of $\Omega_1$, $\Omega_2$ in comparison with $\gamma$ (at the end of the calculation, we put $\varepsilon=1$). Then we can write the solution to equation (19) and light pressure force equation (9) in the form

$$\begin{aligned}\varrho_{11}&=\sum_{n=0}^{4}\varepsilon^n\varrho_{11}^{(n)},\quad \varrho_{22}=\sum_{n=0}^{4}\varepsilon^n\varrho_{22}^{(n)},\\\sigma_{12}&=\sum_{n=0}^{3}\varepsilon^n\sigma_{12}^{(n)},\quad f=\sum_{n=0}^{4}\varepsilon^nf^{(n)}.\end{aligned} \quad (21)$$

From equation (19) it is easy to see that

$$\begin{aligned}&\varrho_{11}^{(0)}=1,\quad \varrho_{11}^{(1)}=0,\quad \varrho_{22}^{(0)}=0,\quad \varrho_{22}^{(1)}=0,\\&\sigma_{12}^{(0)}=0,\quad f^{(0)}=0.\end{aligned} \quad (22)$$

The first order in $\varepsilon$ of equation (19) gives

$$\begin{aligned}\frac{\partial\sigma_{12}^{(1)}}{\partial t}&=\frac{i}{2}\Omega_1e^{-i\delta t-ikz}+\frac{i}{2}\Omega_2e^{-i\delta t+ikz}-\frac{\gamma}{2}\sigma_{12}^{(1)},\\\frac{\partial\sigma_{21}^{(1)}}{\partial t}&=-\frac{i}{2}\Omega_1^*e^{i\delta t+ikz}-\frac{i}{2}\Omega_2^*e^{i\delta t-ikz}-\frac{\gamma}{2}\sigma_{21}^{(1)}.\end{aligned} \quad (23)$$

The solution to equation (23) is

$$\begin{aligned}\sigma_{12}^{(1)}(t)&=\sigma_{12}^{(1)}(0)+\frac{i}{2}\int_0^t e^{\frac{1}{2}\gamma(t'-t)}\left(\Omega_1(t_1')e^{-i\delta t'-ikz'}\right.\\&\quad\left.+\Omega_2(t_2')e^{-i\delta t'+ikz'}\right)dt',\\\sigma_{21}^{(1)}(t)&=\sigma_{21}^{(1)}(0)-\frac{i}{2}\int_0^t e^{\frac{1}{2}\gamma(t'-t)}\left(\Omega_1^*(t_1')e^{i\delta t'+ikz'}\right.\\&\quad\left.+\Omega_2^*(t_2')e^{i\delta t'-ikz'}\right)dt',\end{aligned} \quad (24)$$

where $z'$ is the coordinate of the atom at time $t'$, $t'_1 = t' - z'/c$, $t'_2 = t' + z'/c$. Substituting equations (24) into (9) and taking into account equation (20), we find the light pressure force of in the second order in the field:

$$f^{(2)} = -\frac{i}{2}\hbar k\left(\Omega_1^* \sigma_{12}^{(1)} e^{i\delta t + ikz} - \Omega_1 \sigma_{21}^{(1)} e^{-i\delta t - ikz}\right) + \frac{i}{2}\hbar k\left(\Omega_2^* \sigma_{12}^{(1)} e^{i\delta t - ikz} - \Omega_2 \sigma_{21}^{(1)} e^{-i\delta t + ikz}\right). \quad (25)$$

Let an atom moves with a velocity $v$. Then in a small time interval we can assume that $z = z_0 + vt$, $z' = z_0 + vt'$. Substituting (24) in (25) and averaging the force over the wavelength, we find the value of the steady-state force that is achieved when $t \gg \gamma^{-1}$. After averaging over the modulation period of $2\pi/\Omega_m$, it becomes

$$\tilde{f}^{(2)} = \frac{1}{2}\,\mathrm{Re}\sum_{n=-\infty}^{\infty} \frac{\hbar k \Omega_R^2 J_n^2(\beta)}{\frac{1}{2}\gamma - i\left[\delta - n\Omega_m + (k + n\kappa)v\right]} - \frac{1}{2}\,\mathrm{Re}\sum_{n=-\infty}^{\infty} \frac{\hbar k \Omega_R^2 J_n^2(\beta)}{\frac{1}{2}\gamma - i\left[\delta - n\Omega_m - (k + n\kappa)v\right]} \quad (26)$$

where $\kappa = \Omega_m/c$. As far as usually the ratio $\kappa/k$ is less than $10^{-5}$, we neglect $\kappa/k$ in the following calculations. At a low velocity of the atom, the expression (26) becomes

$$\tilde{f}^{(2)} = -\sum_{n=-\infty}^{\infty} \frac{\hbar k^2 \gamma \Omega_R^2 \delta_n v J_n^2(\beta)}{\left(\frac{1}{4}\gamma^2 + \delta_n^2\right)^2}, \quad (27)$$

where $\delta_n = \delta - n\Omega_m = \omega_0 - \omega - n\Omega_m$ is the detuning of $n$-th spectral component of the FM wave from the frequency of the atomic transition.

If the modulation frequency is high in comparison with the spectral width of the absorption line of the atom, $\Omega_m \gg \gamma$, the light pressure force is mostly determined by the term with a minimum value of $|\delta_n|$ in the sum (27). In this case, the decelerating force occurs when the frequency of the spectral component of the radiation which is closest to the frequency $\omega_0$ is less than $\omega_0$ (red detuning). This is a well-known result in the theory of the Doppler cooling of atoms by a monochromatic standing wave [7].

From equations (19), we find the equation for the population of the excited state in the second order in $\varepsilon$:

$$\frac{\partial}{\partial t}\varrho_{22} = -\frac{i}{2}\sigma_{12} e^{i\delta t}\left(\Omega_1^* e^{ikz} + \Omega_2^* e^{-ikz}\right) + \frac{i}{2}\sigma_{21} e^{-i\delta t}\left(\Omega_1 e^{-ikz} + \Omega_2 e^{ikz}\right) - \gamma\varrho_{22}. \quad (28)$$

Solving equation (28) with the non-diagonal density matrix elements given by equations (24), we find the population of the excited state $\varrho_{22}^{(2)}$ of a slow ($kv \ll \gamma$) atom. We are interested in the averaged over the wavelength steady-state value of the light pressure force that is achieved when $t \gg \gamma^{-1}$. After additional averaging over the modulation period of $2\pi/\Omega_m$, the population becomes

$$\tilde{\varrho}_{22}^{(2)} = \frac{1}{2}\sum_{n=-\infty}^{\infty} \frac{\Omega_R^2 J_n^2(\beta)}{\frac{1}{4}\gamma^2 + \delta_n^2}. \quad (29)$$

Now we know the rate of spontaneous emission of photons $\gamma\tilde{\varrho}_{22}$, and from this we can calculate the momentum diffusion coefficient [33]

$$D_p = \frac{1}{2}(\hbar k)^2 (1 + Q + \xi)\gamma\tilde{\varrho}_{22}, \quad (30)$$

which quantifies the time dependence of the mean-squared momentum deviation $p$ from its mean value $\langle p\rangle$,

$$\langle (p - \langle p\rangle)^2\rangle = 2D_p t. \quad (31)$$

In equation (30) the Mandel parameter $Q$ takes into account the non-Poisson statistics of scattered photons. For the intensities considered here, it is small and we neglect $Q$ in the following calculations. Parameter $\xi$ is determined by the angular distribution of scattered photons. For an one-dimensional model that we study, when photons can scatter in two opposite directions along the propagation of the light waves, $\xi = 1$. It is for this model the well-known formula for the minimal temperature of atoms in the field of the standing wave

$$T_D = \frac{\hbar\gamma}{2k_B}, \quad (32)$$

is valid. Here $k_B$ is the Boltzmann constant.

Taking into account that at low atomic velocity, $v \ll \gamma/k$, the decelerating force, according to equation (27), equals

$$\tilde{f}^{(2)} = -\alpha v, \quad (33)$$

where

$$\alpha = \sum_{n=-\infty}^{\infty} \frac{\hbar k^2 \gamma \Omega_R^2 \delta_n J_n^2(\beta)}{\left(\frac{1}{4}\gamma^2 + \delta_n^2\right)^2}, \quad (34)$$

we calculate the temperature of the atomic ensemble [33]

$$T = \frac{D_p}{\alpha k_B} = \frac{\hbar}{2k_B}\frac{\sum_{n=-\infty}^{\infty} J_n^2(\beta)\left(\frac{1}{4}\gamma^2 + \delta_n^2\right)^{-1}}{\sum_{n=-\infty}^{\infty} \delta_n J_n^2(\beta)\left(\frac{1}{4}\gamma^2 + \delta_n^2\right)^{-2}}, \quad (35)$$

which is defined for positive values of $T$. In the case $\beta = 0$, only Bessel function of zero order gives a contribution to equation (35) and the temperature reaches the minimum value equation (32) at

$$\delta = \delta_{opt} = \frac{\gamma}{2}. \quad (36)$$

Minimal temperature equation (32) is also achieved in $\beta \neq 0$ case provided the condition $\Omega_m \gg \gamma$ is satisfied and the detuning $\delta_n$ of the corresponding spectral component of FM waves from the frequency of the atomic transition is close to equation (36) (of course, if the relative intensity of this component $J_n^2(\beta)$ is not very small). In the case $\Omega_m/\gamma \leq 1$ we cannot derive the analytical expression for the detunings $\delta$ which correspond to the local minima of equation (35). We should note that it is this ratio of the modulation frequency and the rate of spontaneous emission, which expands the range of the velocity where an

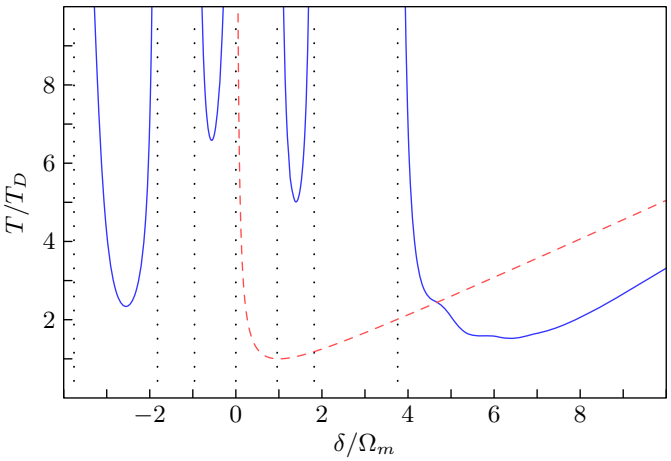


**Fig. 2.** An example of the dependence of the temperature of the ensemble of "heavy" atoms on the detuning $\delta$ in the field of counter-propagating FM waves, calculated from equation (35) for $\gamma = 2\pi \times 10$ MHz, $\Omega_m = 2\pi \times 5$ MHz. The modulation index is $\beta = 5$ (solid curves) and $\beta = 0$ (dashed curve). Vertical asymptotes of the depicted curves are marked with dots.

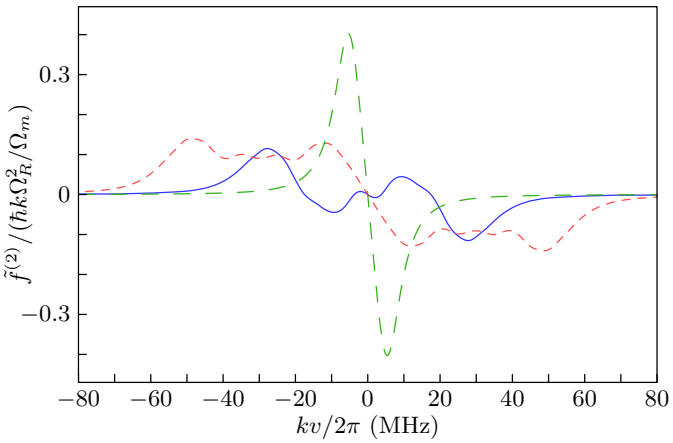


**Fig. 3.** Dependencies of the light pressure force exerted on a "heavy" atom in the field of counter-propagating FM waves on the Doppler frequency shift $kv$, calculated from equation (26). Parameters: $\gamma = 2\pi \times 10$ MHz, $\Omega_m = 2\pi \times 5$ MHz for all curves, $\beta = 5$, $\delta = 2\pi \times 8$ MHz (solid curve), $\beta = 5$, $\delta = 2\pi \times 30$ MHz (dashed curve), $\beta = 0$, $\delta = 2\pi \times 5$ MHz (long-dashed curve).

atom effectively interacts with the field, was proposed to use for the collimation of an atomic beam [27].

Our calculation of the temperature of an atomic ensemble is based on the assumption of an approximately linear dependence of the light pressure force on the atomic velocity near $v = 0$. As far as velocities of atoms lie mainly in the range $[-v_{rms}, v_{rms}]$, where $v_{rms} = \sqrt{k_B T/m}$, after calculation the temperature according to equation (35) we should check how the velocity dependence of the light pressure force (26) is close to linear within the specified limits. Velocity range in which we check the consistency of calculations narrows with increase of mass of the atom, so we can say that equation (35) is valid for "heavy atoms".

An example of the dependence of the temperature of the ensemble of "heavy" atoms on the detuning $\delta$ of the carrier frequency of the FM waves from the transition frequency of the atom according to equation (35) is shown in Figure 2. The curves are separated by horizontal gaps. These gaps arise from the oddness of function equation (35) with respect to $\delta$. Indeed, if $T > 0$ for $\delta = \delta_1$, then equation (35) gives $T < 0$ for $\delta = -\delta_1$. Therefore, if the atomic ensemble is characterized by some temperature for $\delta = \delta_1$, it cannot be described by temperature for $\delta = -\delta_1$. The dashed curve in Figure 2 shows the corresponding dependence for the case of the monochromatic waves ($\beta = 0$).

Let us check the consistency of the calculation of the temperature from equation (35) for sodium atoms. We choose two detunings at which the temperature is close to the local minima for shown in Figure 2 dependencies, $\delta/\Omega_m = 6$ ($\delta/2\pi = 30$ MHz) and $\delta/\Omega_m = 1.6$ ($\delta/2\pi = 8$ MHz). Figure 3 presents the dependencies of the light pressure force on the velocity for these detunings. The long-dashed curve in the figure shows the corresponding dependence for the case of the monochromatic waves and $\delta/\Omega_m = 1$ ($\delta/2\pi = 5$ MHz). As can be seen, at zero velocity the derivative of the force on the velocity at $\delta/2\pi = 30$ MHz much greater than at $\delta/2\pi = 8$ MHz. In the first case, the range in which the light pressure force is linear on the velocity is much wider. The wavelength of the atomic transition in $^{23}$Na is $\lambda = 589.16$ nm, Doppler cooling limit is $T_D = 237.2\,\mu$K [7]. Simple calculations give $v_{rms} \sim 12$ m s$^{-1}$ for the case $\delta = 2\pi \times 8$ MHz. This greatly exceeds the interval of linearity about $\sim$1.2 m s$^{-1}$. At the same time in the case $\delta = 2\pi \times 30$ MHz we have $v_{rms} \sim 0.8$ m s$^{-1}$, that is significantly lower than the interval of linearity about $\sim$6 m s$^{-1}$. Similar calculation for the case $\delta/2\pi = 5$ MHz and $\beta = 0$ (long-dashed curve in Fig. 3) shows that $v_{rms}$ in this case is much smaller than the width of linear dependence of the light pressure force on velocity near $v = 0$.

Thus, the consistency test for calculation of the atomic ensemble temperature from equation (35) and the condition that this formula is valid, showed that when $\delta = 2\pi \times 30$ MHz a stationary temperature of a sodium atomic ensemble in the field of the counter-propagating FM waves is achieved and, if $\delta = 2\pi \times 8$ MHz, equation (35) is not consistent with the condition of its correctness. Moreover, in the latter case, the formation of two subsets of the ensemble of atoms with velocities close to $\pm 10.25$ m s$^{-1}$ is expected, since at these velocities the light pressure force is zero and its derivative with respect to velocity is negative (see Fig. 3). Thus, we conclude that FM waves can split down an atomic beam into two. Each of these beams is characterized by the mean velocity along the direction of wave propagation and, probably, temperature, calculation of which can be made similar to the above method, linearizing the dependence of the light pressure force on the velocity at near-zero values of the force.

Note that the dependence of light pressure force shown in Figure 3 by solid curve resembles the dependence of light pressure force in a strong standing monochromatic

wave considered in [6,35], where more than one zeros on the dependence of force vs. velocity for some parameters of the atom-field interaction was found. Therefore, we conclude that some phenomena inherent to the atom's interaction with a strong standing wave can manifest themselves in the interaction of atoms with weak counterpropagating FM waves.

To analyze the possible formation of a trap for atoms by the counter-propagating FM waves, we should find the coordinate dependence of the light pressure force. For this purpose, we calculate the force up to the fourth order on $\varepsilon$. The derivation of the force $\tilde{f}^{(4)}$ averaged over the modulation period is similar to the derivation of equation (26). For the resonant case of the interaction of slow ($kv \ll \gamma$) atoms with the field and small modulation index the result is [22,26]

$$\tilde{f}^{(4)} = -\frac{4\hbar k\beta^2\Omega_R^4\Omega_m^4}{\gamma^2 c\left(\Omega_m^2+\frac{\gamma^2}{4}\right)^2}z. \tag{37}$$

This force is written for $z \ll c/\Omega_m$. It is directed towards the center of the trap ($z = 0$) and under favorable conditions (small heating of the atoms due to the momentum diffusion) tries to keep the atoms at $z = 0$, where the counter-propagating waves "collide". The magnitude of the force and its direction depends on the time delay $2z/c$ between the counter-propagating waves in the point where the atom is located that indicates the involvement of both waves in the formation of the restoring force. A simple interpretation of the force might be predominant absorption of a photon of the wave traveling to the center of the trap followed by the stimulated emission of a photon into the counter-propagating wave similar to the force exerted on an atom in the trap formed by the sequences of the counter-propagating laser pulses [8,9,15].

## 5 The state vector of the atom

Suppose that the state vector of the atom $|\psi(t)\rangle$ at time $t$ is normalized to unity. The state vector $|\psi(t+\Delta t)\rangle$ at time $t+\Delta t$ we find in two steps [31].

1. From Schrödinger's equation (15), it follows that after a sufficiently small $\Delta t$ the state vector $|\psi(t)\rangle$ becomes

$$|\psi^{(1)}(t+\Delta t)\rangle = \left(1-\frac{i\Delta t}{\hbar}H\right)|\psi(t)\rangle. \tag{38}$$

Because the Hamiltonian (16) is non-Hermitian, after time step $\Delta t$ the state vector $\psi^{(1)}(t+\Delta t)$ is not normalized to unity. The square of its norm is

$$\langle\psi^{(1)}(t+\Delta t)|\psi^{(1)}(t+\Delta t)\rangle = 1-\Delta P, \tag{39}$$

where

$$\Delta P = \frac{i\Delta t}{\hbar}\langle\psi(t)|H-H^{+}|\psi(t)\rangle = \gamma\Delta t|c_2|^2. \tag{40}$$

2. In the second stage we take into account the possible quantum jump (spontaneous emission of a photon). We introduce the random variable $\epsilon$, which is uniformly distributed between zero and one. If $\epsilon > \Delta P$ (in most cases, since $\Delta P \ll 1$), the jump does not occur. We normalize the state vector of the atom at time $t+\Delta t$ and it becomes

$$|\psi(t+\Delta t)\rangle = \frac{|\psi^{(1)}(t+\Delta t)\rangle}{\sqrt{1-\Delta P}}, \quad \Delta P < \epsilon. \tag{41}$$

If $\epsilon \le \Delta P$, the jump occurs and the wave state vector of the atom reads

$$|\psi(t+\Delta t)\rangle = |1\rangle, \quad \Delta P > \epsilon. \tag{42}$$

Substituting equations (17) and (16) in equation (15) gives

$$\begin{aligned} i\hbar\frac{d}{dt}c_1 &= -\boldsymbol{d}_{12}\boldsymbol{E}c_2e^{-i\omega_0 t},\\ i\hbar\frac{d}{dt}c_2 &= -\boldsymbol{d}_{21}\boldsymbol{E}c_1e^{i\omega_0 t}-i\hbar\frac{\gamma}{2}c_2. \end{aligned} \tag{43}$$

After applying RWA (neglecting rapidly oscillating terms $\sim e^{\pm 2i\omega_0 t}$) [34] to equation (43) with the field described by equation (1) and taking into account the definition of the Rabi frequencies $\Omega_1$, $\Omega_2$ we arrive at

$$\begin{aligned} \frac{d}{dt}c_1 &= -\frac{i}{2}\left(\Omega_1e^{-ikz}+\Omega_2e^{ikz}\right)c_2e^{-i\delta t},\\ \frac{d}{dt}c_2 &= -\frac{i}{2}\left(\Omega_1^*e^{ikz}+\Omega_2^*e^{-ikz}\right)c_1e^{i\delta t}-\frac{\gamma}{2}c_2. \end{aligned} \tag{44}$$

Solving equations (44) with the normalization after each step of integration, we find the time dependence of $c_1$, $c_2$ between quantum jumps. In the case when a quantum jump occurs after the integration step, the probability amplitudes becomes $c_1 = 1$, $c_2 = 0$. The known probability amplitudes allow to calculate the light pressure force exerted on the atom and describe its motion by integrating Schrödinger's and Newton's equations.

## 6 The procedure of numerical calculation

To simulate the atom's motion, we solve equations (7) and (44), where the light pressure force is given by equation (9) and the non-diagonal matrix elements are defined by equation (18). To describe the motion of the atom, we should also take into account a stochastic change of the atomic momentum due to the momentum diffusion process. In the case of low-intensity laser radiation, when the population of the excited state is small, the light pressure force and the coefficient of momentum diffusion are equal to the sum of the quantities resulting from each of the counter-propagating waves [36]. In this case, spontaneous emission occurs after each photon absorption, and the fluctuations of the momentum due to the stimulated processes occur as frequently, as fluctuations due to the spontaneous radiation [6]. In our calculations we assume that the atomic momentum changes by $\pm\hbar k$ with equal probability in the course of spontaneous emission. In the

works [15,17,37] a similar procedure was the basis of computer simulation of the atomic motion in the counter-propagating light waves.

The described consideration of the momentum diffusion process is valid, as noted above, for weak fields $\Omega_R < \gamma$. In the case of high-intensity counter-propagating waves we consider a momentum diffusion (momentum fluctuations) by the similar way, bearing in mind that the result should be treated as an estimation.

In summary, the calculation algorithm of the motion of atoms in the field of the counter-propagating FM waves consists of the following steps:

- Equations (7) and (44) are integrated by the Runge-Kutta method of the fourth order.
- After every step we check whether a quantum jump occurred, and the state vector is normalized.
- If a quantum jump occurred, the velocity of the atom changes by

$$\Delta v = \hbar k(\epsilon_1 - 0.5)/(M|\epsilon_1 - 0.5|) + \hbar k(\epsilon_2 - 0.5)/(M|\epsilon_2 - 0.5|), \quad (45)$$

where $\epsilon_{1,2}$ are random numbers, uniformly distributed over the interval $[0, 1]$.

One of the random numbers is responsible for the fluctuation of the momentum due to a spontaneously emitted photon, and the other is responsible for the fluctuation of the momentum resulting from fluctuations of absorption and stimulated emission of photons. As far as we use quasi-classical approach, the simultaneous consideration of both sources of the momentum fluctuations in equation (45) does not affect the statistical characteristics of the atomic ensemble. More details concerning fluctuations in the one-dimensional model of spontaneous emission are discussed in [37].

## 7 Results and discussion

We simulate the behaviour of sodium $^{23}$Na and cesium $^{133}$Cs atoms in FM field, in which the cyclic interaction of the atoms with a laser radiation can be realized [7]. The relevant parameters of the atom-field interaction are following. The wavelength for the transition $3^2S_{1/2}$–$3^2P_{3/2}$ in a sodium atom is $\lambda = 589.16$ nm, the spontaneous emission rate is $\gamma = 2\pi \times 10$ MHz, the Doppler cooling limit is $T_D = 237.2$ $\mu$K [7]. The wavelength for transition $6^2S_{1/2}$–$6^2P_{3/2}$ in a cesium atom is $\lambda = 852.35$ nm, the spontaneous emission rate is $\gamma = 2\pi \times 5.18$ MHz, the Doppler cooling limit is $T_D = 124.39$ $\mu$K [7].

### 7.1 Low-intensity counter-propagating waves

When the counter-propagating waves are weak ($\Omega_R^2 \ll \gamma^2$), the coordinate dependent part of the light pressure force $\tilde{f}^{(4)}$ (37) is small in comparison with the part $\tilde{f}^{(2)}$ (26) of the light pressure force which depends only on the atom's velocity. Thus, using fields of low intensity we can control predominantly the velocity distribution of atoms. At a high modulation frequency, $\Omega_m \gg \gamma$, when the atoms interact mainly with the closest to $\omega_0$ spectral component of the field, their movement is essentially the same as the well-studied [6,7] motion of atoms in the field of the monochromatic standing wave. In the case $\Omega_m \geq \gamma$, the atoms interact with several spectral components of the field, that greatly extends the interval of their velocities in which the atom-field interaction is significant. This circumstance is the basis for the proposal of using "white light" to control the atomic motion [18,21,27]. In this subsection, we calculate the temperature of the degree of freedom of the atoms along the direction of the FM waves propagation and consider some aspects of the motion of atoms in such waves.

In Section 4 we have shown that the equation (35) for the temperature of the atoms is valid when $v_{rms}$ lays in the interval in which the force exerted on the atom is approximately proportional to its velocity. In this case $v_{rms} \propto M^{-1/2}$, and we conclude that the equation (35) is valid if the mass of atom is large enough ("heavy atom" model). Strictly speaking, a priory we don't know for each specific case if equation (35) gives the right result. The verification procedure is the following. We calculate the temperature of the atomic ensemble according to equation (35). Then we find $v_{rms}$ and check if the light pressure force linearly depends on the atomic velocity, as it was described in Section 4. In case of a positive answer equation (35) gives the right result. In this section we check equation (35) by comparing it with the temperature found from the numerical calculation of the statistical parameters of atoms (including $v_{rms}$). We show that for some parameters of the atom-field interaction the expression equation (35) is correct for Na and Cs, and for other parameters it gives correct result only for Cs atoms. Figure 4 illustrates this. We see that for detunings corresponding to the second and the third curves in Figure 4 we cannot talk about the temperature of the ensemble of sodium atoms, but can talk about the temperature of the cesium atoms. The reason is very simple. The cesium atomic mass is much greater than the sodium atomic mass, and the rate of spontaneous emission of a cesium atom is a half of the rate of spontaneous emission of a sodium atom. As a result, the light pressure force exerted on a cesium atom almost linearly depends on its velocity in the interval $[-v_{rms}, v_{rms}]$, unlike the light pressure force exerted on a sodium atom. This is illustrated by Figure 5, which shows the dependence of the light pressure force exerted on sodium and cesium atoms on their velocity for parameters of Figure 4 and $\delta/\Omega_m = 1.6$ (the circle on the third curve in the figure).

As can be seen in Figure 5, the velocity distribution of sodium atoms after 10 ms of the atom-field interaction is much wider and the corresponding distribution of cesium atoms is much narrower than the interval of almost linear dependence of the light pressure force on the velocity of atoms (compare (a) with (b) and (c) with (d) in Fig. 5). It should be noted that the distribution of cesium

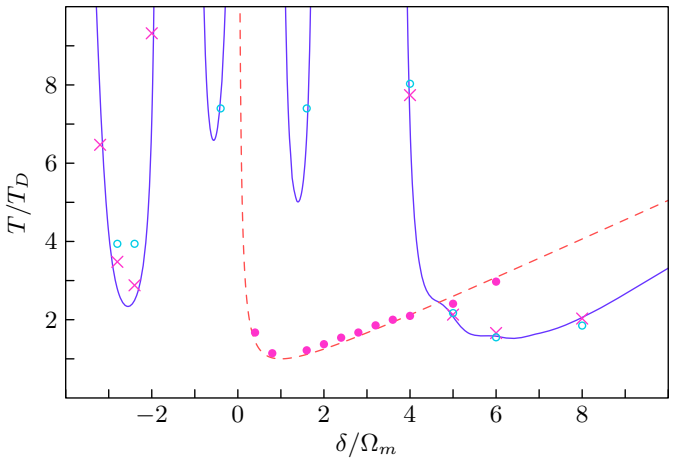


**Fig. 4.** The dependence of the temperature of the atomic ensemble of 100 $^{133}$Cs atoms (circles) and 100 $^{23}$Na atoms (crosses and full circles) in units of the Doppler cooling limit temperature $T_D$ on the carrier frequency detuning of the FM wave from the atomic transition frequency in units of the modulation frequency. Solid and dashed curves display calculation by equation (35), crosses, circles and full circles are results of the numerical calculations described in Section 6. Parameters: $\Omega_m = 0.5\gamma$, $\Omega_R = 2\pi \times 2$ MHz and $\beta = 5$ (solid curve, crosses and circles), $\beta = 0$ (dashed curve and full circles).

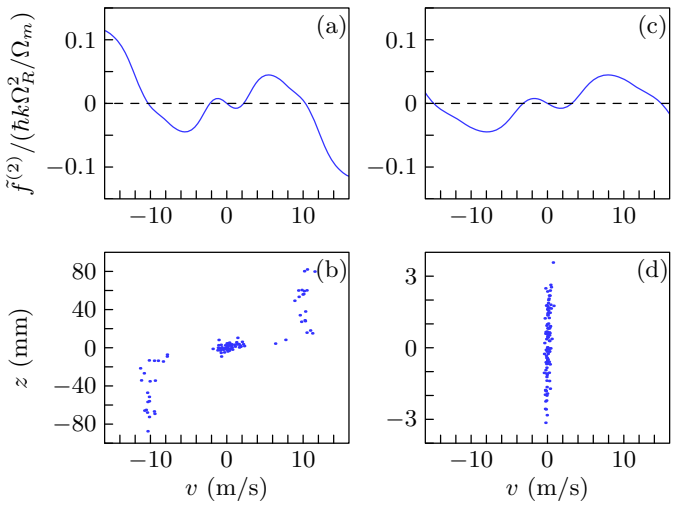


**Fig. 5.** The dependence of the light pressure force exerted on sodium (a) and cesium (c) atoms on their velocity in the field of the counter-propagating FM waves and the corresponding distribution of 100 sodium (b) and cesium (d) atoms with zero initial velocity in the phase plane after 10 ms of their interaction with the field. Parameters are $\Omega_m = 0.5\gamma$, $\beta = 5$, $\Omega_R = 2\pi \times 2$ MHz, $\delta/\Omega_m = 1.6$.

atoms in the phase plane, shown in Figure 5d, is formed after 4 ms of the interaction of atoms with the field while the distribution of sodium atoms after 10 ms is not yet established; the sodium atoms are moving from the area near zero velocity to the areas of about 10 m s$^{-1}$ velocity. This phenomenon is due to the phase modulation of the counter-propagating waves; in the field of waves with the same parameters but $\beta = 0$, according to our calculations, sodium atoms moves with close to zero velocity with $\Delta v = 0.32$ m s$^{-1}$.

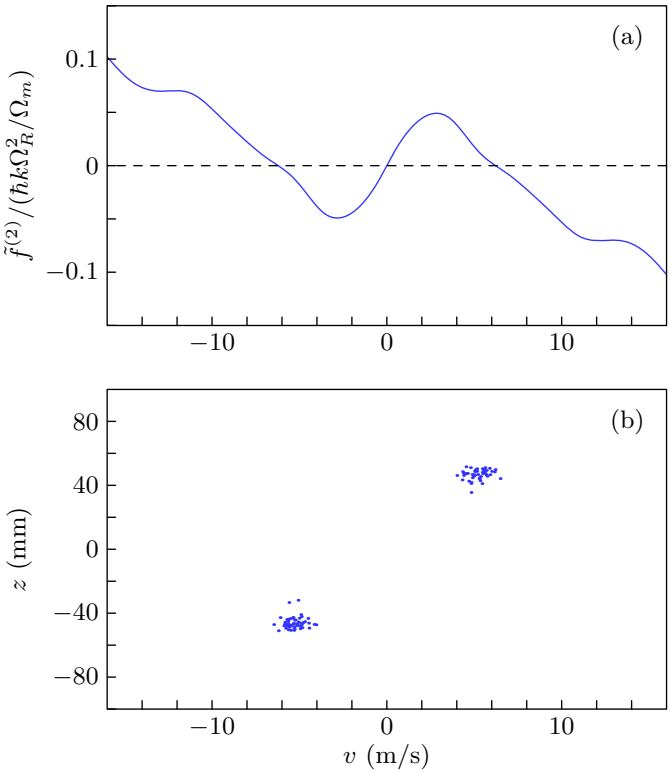


**Fig. 6.** The dependence of the light pressure force exerted on sodium atoms on their velocity in the field of the counter-propagating FM waves (a) and the corresponding distribution of 100 sodium atoms with zero initial velocity in the phase plane after 10 ms of their interaction with the field (b). Parameters: $\Omega_m = 2\pi \times 5$ MHz, $\beta = 5$, $\Omega_R = 2\pi \times 2$ MHz, $\delta = 2\pi \times 14$ MHz.

There are intervals in which the temperature of the atomic ensemble in Figures 2 and 4 is not defined (formula (35) yields a negative value), because the derivative of the light pressure force with respect to the velocity at $v = 0$ is positive. However, the negative derivative of the force can be achieved at non-zero velocities (denote them by $\tilde{v}_n$, $n = 1, 2$), as shown in Figure 6a. When the velocity of the atom deviates from the value that corresponds to the zero force, e.g., $\tilde{v}_1$, the force tries to return the velocity to $\tilde{v}_1$. Fluctuations of the velocity due to the spontaneous emission lead to the formation of an ensemble of atoms with close to $\tilde{v}_1$ velocity. If the initial velocity of all atoms in the ensemble is zero, two subensembles consisting of approximately equal amounts of atoms will be formed at velocities which are close to $\tilde{v}_1$, $\tilde{v}_2$ (see Fig. 6b) with $\Delta v = 0.52$ m s$^{-1}$ that corresponds to $T \approx 3.1 T_D = 740$ mK. For comparison, we also modelled the motion of atoms in the field of counterpropagating waves with the same parameters but $\beta = 0$. According to our calculations, all sodium atoms moves with close to zero velocity with $\Delta v = 0.38$ m s$^{-1}$ and the length of the atomic cloud is ∼2 mm at $t = 10$ ms.

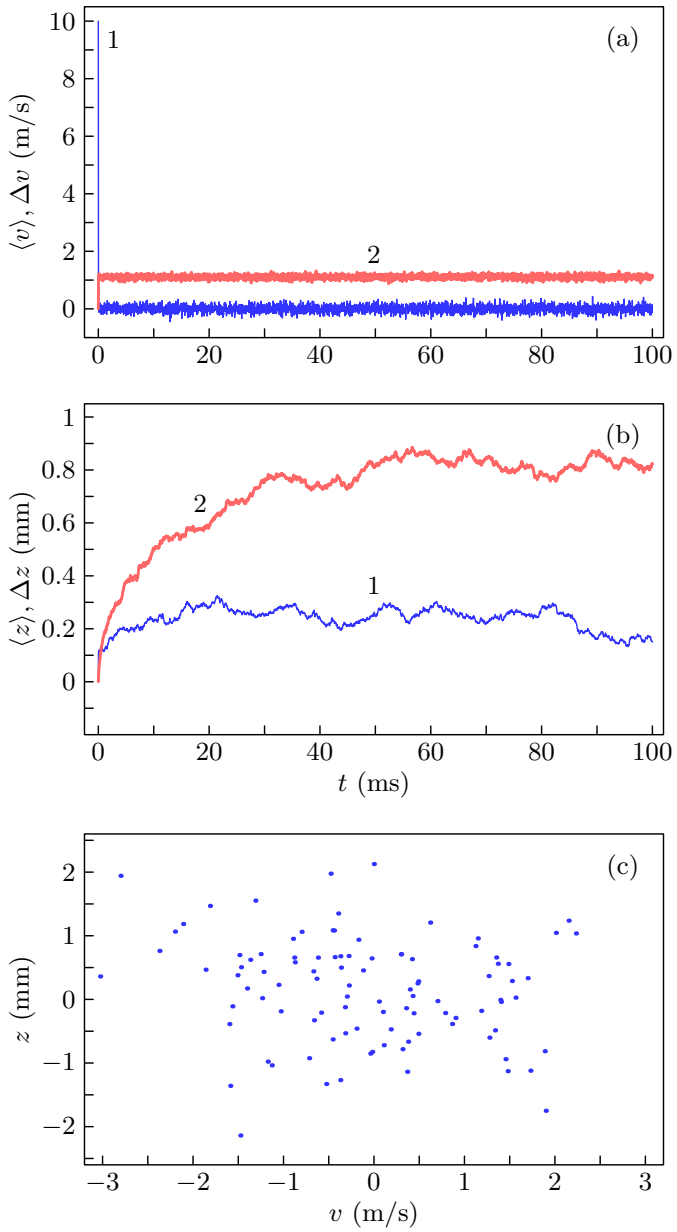


**Fig. 7.** The time dependencies of (a) the mean velocity of $^{23}$Na atoms (1) and the standard deviation of the atomic velocities from the mean velocity (2), (b) the mean coordinates of the atoms (1) and the standard deviation of the atomic coordinates from the mean coordinate (2) calculated for 100 sodium atoms with an initial velocity of 10 m s$^{-1}$. (c) The distribution of atoms in the phase plane after 100 ms of their interaction with the field. Parameters: $\Omega_m = 2\pi \times 50$ MHz, $\beta = 0.5$, $\Omega_R = 2\pi \times 50$ MHz, $\delta = 2\pi \times 25$ MHz.

Figure 6 shows that the counter-propagating FM waves can split a collimated atomic beam into two roughly equal parts if the FM waves travel perpendicular to the motion of the atoms in the beam. Obviously, it is not difficult to group almost all atoms near $\tilde{v}_1$ or $\tilde{v}_2$. For this purpose, the angle between the direction of propagation of laser radiation and the direction of motion of the atoms in the collimated beam should be a little different from 90°.

### 7.2 High-intensity counter-propagating FM waves

When the intensity of the counter-propagating FM waves becomes high, the field may confine atoms. Figure 7 demonstrates this. Sodium atoms start to move with the initial velocity of 10 m s$^{-1}$ in the point with coordinate $z = 0$. The FM waves quickly decelerate them (within 50 $\mu$s). The velocity distribution of the atoms becomes stationary with the root-mean-square deviation of the atomic velocities from zero value of $\Delta v = 1.1$ m s$^{-1}$, which corresponds to the temperature of the atomic ensemble 3.2 mK. The size of the atomic cloud is stabilized after about 50 ms. The root-mean-square deviation $\Delta z$ of the coordinates of atoms from the mean coordinate $\langle z \rangle$ is about 0.8 mm. The distribution of atoms in the phase plane, shown in part (c) of the figure, is approximately uniform over the coordinate within the interval $\pm 1.5 \Delta z$ and over the velocity within the interval $\pm 1.5 \Delta v$ that confirms the applicability of the concept of temperature to the atomic ensemble. For comparison, we also modelled the motion of atoms in the field of counterpropagating waves with the same parameters but $\beta = 0$. According to our calculations, all sodium atoms moves with close to 2.4 m s$^{-1}$ velocity with $\Delta v = 0.31$ m s$^{-1}$ and quickly leave the vicinity of $z = 0$ (after 10 ms their average coordinate is 24 mm).

Figure 6 shows a situation when atoms with close to zero velocity are absent in the ensemble. In this case, we cannot use $\Delta v$ to estimate the temperature of atoms in the ensemble. Figure 8 shows the temporal evolution of the mean velocity and the mean coordinate of the atomic ensemble and the standard deviations of velocities and coordinates of the atoms from these values, as well as the distribution of atoms in the phase plane after 100 ms of their interaction with the field for a similar situation. The velocity distribution of atoms, in this case, is very far from Maxwell's, therefore we cannot talk about the temperature of the atomic ensemble. The standard deviation of the atomic velocities from the mean velocity reaches a stationary value after about 5 ms of the atom-field interaction, and the mean velocity within about 2 ms reduces to zero and then varies approximately in the range $\pm 1$ m s$^{-1}$ (part (a) of Fig. 8). As far as the main part of atoms are distributed close to velocities $v_c = +3$ m s$^{-1}$ and $-v_c = -3$ m s$^{-1}$, one would think that the counter-propagating FM waves split the atomic ensemble into two sub-ensembles (part (c) of Fig. 8). But in this case, the root-mean-square deviation of coordinates from $z = 0$ would increase with time. Nevertheless, it remains, beginning from about 5 ms, approximately at the same level. We explain this by a change of the velocity of the atoms between $\sim -v_c$ m s$^{-1}$ and $\sim +v_c$ m s$^{-1}$ from time to time. This transition is not instantaneous, as is confirmed by a small number of atoms in the vicinity of zero velocity in Figure 8c. From our point of view, the light pressure force exerted on atoms is close to zero at velocities $\pm v_c$. As a result, the atoms move mostly with almost constant velocity. From time to time, the velocity of such atoms changes due to the momentum diffusion process. Sometimes momentum fluctuations change the sign of the velocity. Then the light pressure force changes the sign and the

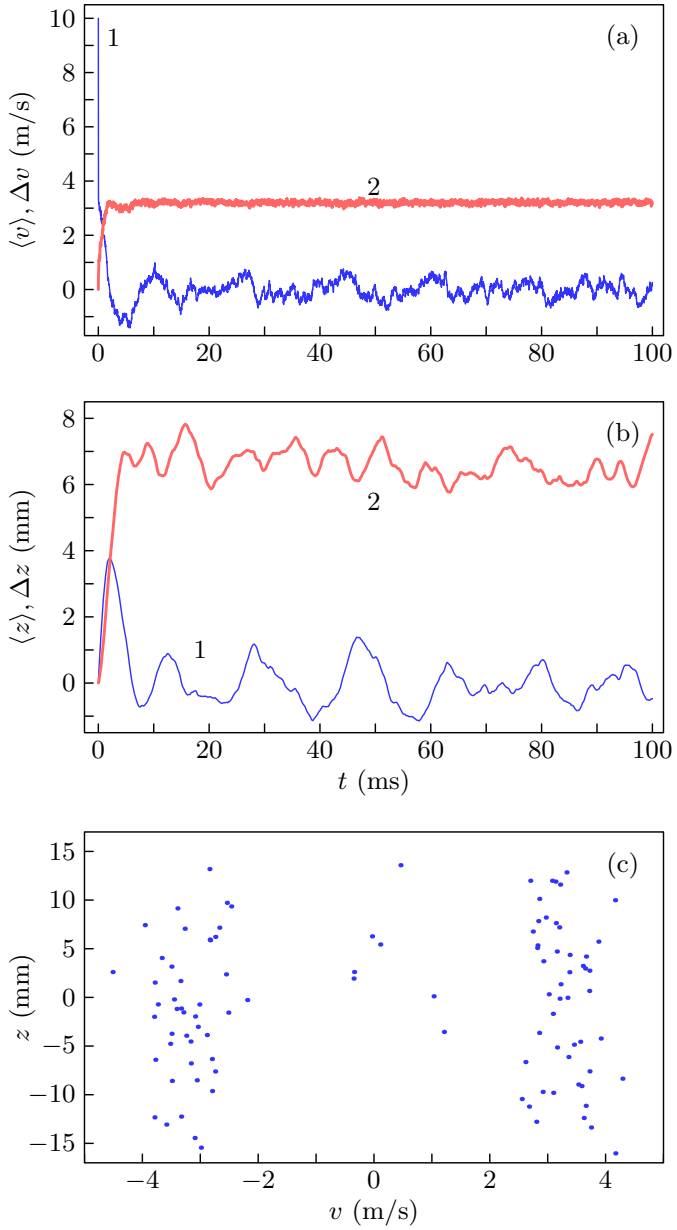


**Fig. 8.** The time dependencies of (a) the mean velocity of $^{23}$Na atoms (1) and the standard deviation of the atomic velocities from the mean velocity (2), (b) the mean coordinate of the atoms (1) and the standard deviation of the atomic coordinates from the mean coordinate (2) calculated for 100 sodium atoms with the initial velocity of 10 m s$^{-1}$. (c) The distribution of atoms in the phase plane after 100 ms of their interaction with the field. Parameters: $\Omega_m = 2\pi \times 50$ MHz, $\beta = 0.5$, $\Omega_R = 2\pi \times 50$ MHz, $\delta = 2\pi \times 35$ MHz.

atomic velocity tends to the other almost constant value ($+v_c \to -v_c$ and vice versa). We also examined the motion of atoms in the field of counterpropagating waves with the same parameters but $\beta = 0$. According to our calculations, all sodium atoms moves with close to 3.2 m s$^{-1}$ velocity with $\Delta v = 0.33$ m s$^{-1}$ and quickly leave the vicinity of $z = 0$ (after 10 ms their average coordinate is 32 mm).

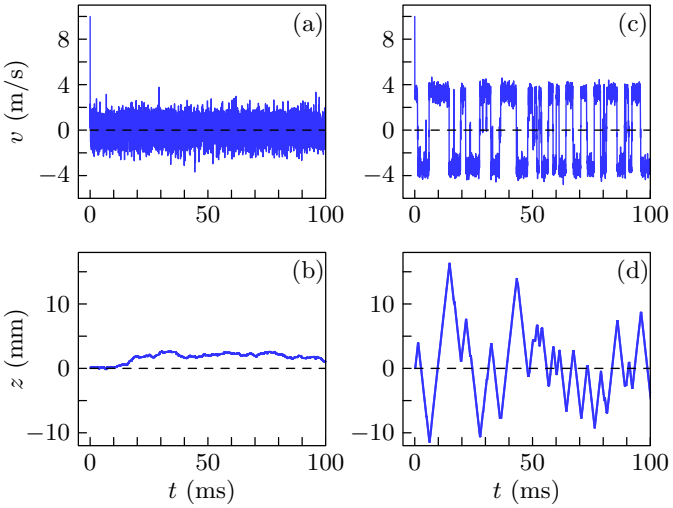


**Fig. 9.** The time dependences of the velocity and the coordinate of a sodium atom for $\delta = 2\pi \times 25$ MHz (a, b) and $\delta = 2\pi \times 35$ MHz (c, d). Parameters: $\Omega_m = 2\pi \times 50$ MHz, $\beta = 0.5$, $\Omega_R = 2\pi \times 50$ MHz.

Figure 9 illustrates the motion of a trapped atom for parameters corresponding to Figure 7, where the temperature of atoms is correctly defined, and Figure 8, where the atoms, though confined in the trap, cannot be characterized by temperature. We see that in the first case (Figs. 9a and 9b) the movement of the atom looks like diffusion. In the second case (Figs. 9c and 9d) the motion of the atoms consists of pieces of almost uniform motion with the change of the sign of the velocity during a very short time.

Figure 10 shows examples of the dependence of the temperature of sodium atoms on the detuning of the carrier frequency of FM waves from the transition frequency of the atom. The temperature of the atoms is defined by the formula

$$\frac{1}{2}k_B T = \frac{1}{2}m\langle v^2\rangle, \tag{46}$$

where $\langle v^2\rangle$ is the mean square of the velocity of atoms. The stationary (up to fluctuations) value of $\langle v^2\rangle$ is achieved after 20–50 $\mu$s. Doppler cooling limit corresponds to the square root of the mean squared velocities of atoms $v_D = 29.47$ cm s$^{-1}$ [7].

Equation (46) gives the temperature of atoms in the case of the Maxwell's velocity distribution. In other cases the "temperature" given by equation (46) refers to the average kinetic energy of the atoms. In particular, the velocity distributions of atoms for $\beta = 0.5$, $\Omega_R/2\pi = 50$ MHz in the range of detuning of 35–45 MHz and for $\beta = 0.5$, $\Omega_R/2\pi = 20$ MHz in the range of detuning of 15–45 MHz are far from Maxwell's distribution function. In some cases, the velocity distribution function is symmetric with respect to zero velocity and has two maxima, like demonstrated in Figure 8c.

We cannot find the temperature of atoms if their mean squared velocity continuously grows. For example, we cannot calculate the temperature of atoms for $\beta = 0.5$, $\Omega_R/2\pi = 20$ MHz and detuning to the left of the first

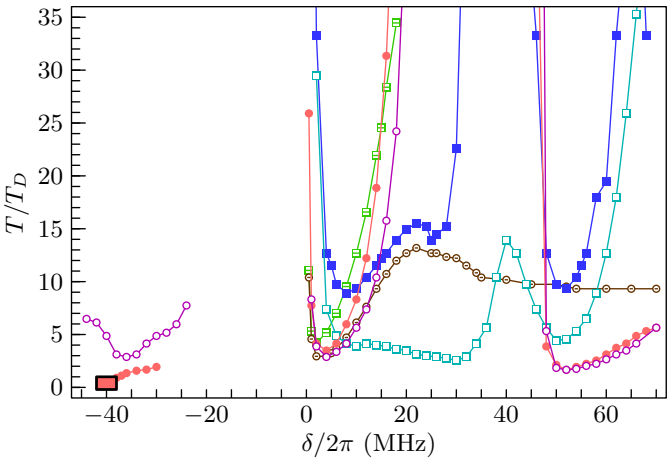


**Fig. 10.** The dependence of the temperature of sodium atoms on the detuning $\delta$ of the carrier frequency $\omega$ of the counter-propagating FM waves from the transition frequency $\omega_0$ of the atom. Parameters: $\Omega_m = 2\pi \times 50$ MHz, $\beta = 0$ (⊟, ⊖), $\beta = 0.5$ (■, •), $\beta = 1$ (□, ○), $\Omega_R = 2\pi \times 50$ MHz (■, ⊟, □), $\Omega_R = 2\pi \times 20$ MHz (•, ○, ⊖). The temperature is calculated for 100 sodium atoms with the initial velocity of 5 m s$^{-1}$. The rectangular near $\delta = -2\pi \times 40$ MHz marks the interval of detuning where the temperature is lower than the Doppler cooling limit.

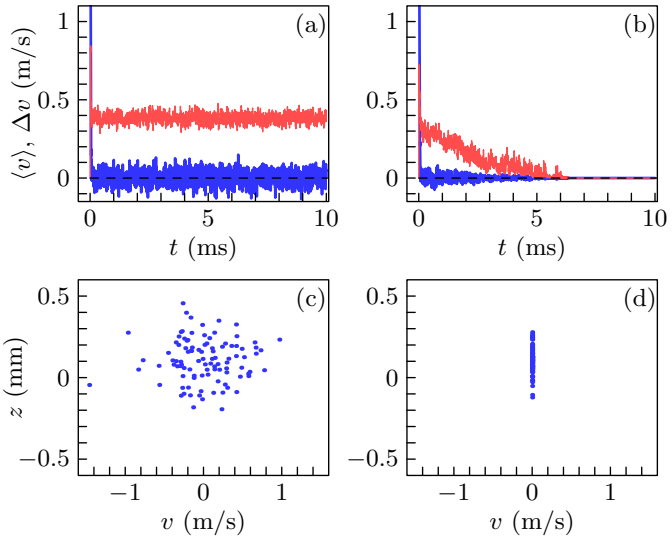


**Fig. 11.** The time dependence of the mean velocity (lower curve, blue online) and standard deviation of velocity (upper curve, red online) from the mean value for an ensemble of 100 sodium atoms in the field of counter-propagating FM waves. The detuning of the carrier frequency from the transition frequency of the atom is $\delta = -2\pi \times 32$ MHz (a) and $\delta = -2\pi \times 38$ MHz (b). The appropriate distributions of atoms in the phase plane after 10 ms from the beginning of the movement are shown in parts (c) and (d) of the figure. Other parameters of the interaction of atoms with the field are $\Omega_m = 2\pi \times 50$ MHz, $\beta = 0.5$, $\Omega_R = 2\pi \times 20$ MHz, the initial velocity of the atoms is $v_0 = 5$ m s$^{-1}$.

point on the plot, as far as almost all atoms have a sub-Doppler temperature but a small number of them accelerates to the velocity of about 100 m s$^{-1}$ after 10 ms of their interaction with the field. Except for a small interval $-41$ MHz $\leq \delta/2\pi \leq -38$ MHz for $\beta = 0.5$ and $\Omega_R/2\pi = 20$ MHz, the temperature of atoms in the field of FM waves for the parameters of the atom-interaction corresponding to Figure 10 exceeds the Doppler cooling limit $T_D$. A probable cause of the unexpectedly low temperature in the region of $\delta$, marked with a rectangular near $\delta = -2\pi \times 40$ MHz, we discuss below.

Figure 11 shows the time dependence of the mean velocity and the root-mean-square deviation of the velocities of atoms from the mean velocity in cases where the temperature of the atomic ensemble is higher than the Doppler cooling limit (a) and lies below it (b). In part (a) of Figure 11 the time dependence of $\Delta v$ achieves the mean value of around 0.38 m s$^{-1}$ and fluctuates near it with a small amplitude, which allows estimating the temperature of an atomic ensemble of about $1.7T_D$. For part (b) of the figure our calculations give $\Delta v$ which is far below the recoil velocity $v_r = \hbar k/m$. It means that the de Broglie wavelength of atoms exceeds $\lambda$ and quasiclassical approach is not valid. Nevertheless, obtained results clearly show that the temperature of atoms is substantially lower than the Doppler cooling limit (in the opposite case we would find $\Delta v > v_r$).

Parts (c) and (d) of Figure 11 show distributions of atoms in the phase plane, corresponding to parts (a) and (b). In part (c) of Figure 11 the velocity distribution of atoms is close to Maxwell's distribution function. Noticeable displacement of the center of mass of the atoms in the positive direction is associated with their initial velocity. In part (d) of Figure 11 the velocity distribution of atoms is obviously not Maxwell's. Here the velocity of atoms lays below $v_r$ that means that quantum treatment of atomic motion is needed. Slow atoms are grouped near the nodes of the standing wave that is formed by the counter-propagating FM waves, and predominantly occupy the ground state. The position of the nodes are described by the expression $z_n = \frac{1}{2}\lambda n + \frac{1}{4}\lambda$ where $n$ is an integer.

Note that when $\beta = 1$, $\Omega_R/2\pi = 20$ MHz and detuning lies in the interval $-40$ MHz $\leq \delta/2\pi \leq -34$ MHz, a part of atoms are grouped near zero velocity, as illustrated above for the case $\beta = 0.5$ and $\Omega_R/2\pi = 20$ MHz, but their quantity is insufficient for reducing the temperature of atoms below the Doppler cooling limit (see Fig. 10).

Figure 12 shows examples of time dependencies of the velocity of a sodium atom used in the calculation of parts (a) and (b) of Figure 11. When the detuning of the carrier frequency from the transition frequency of the atom equals $\delta = -2\pi \times 32$ MHz, the atom quickly decelerates during 35 $\mu$s (Fig. 12a) and then its velocity oscillates predominantly in the range of $\pm 0.5$ m s$^{-1}$. In this case, the velocity distribution of atoms is close to Maxwell's (see Fig. 11c). When the detuning is $\delta = -2\pi \times 38$ MHz (parts (b) and (d) of the figure), the situation is radically different. Although the atom decelerates during approximately the same time, later the velocity of the atom decreases to the value of about 0.1 m s$^{-1}$ and stochastic

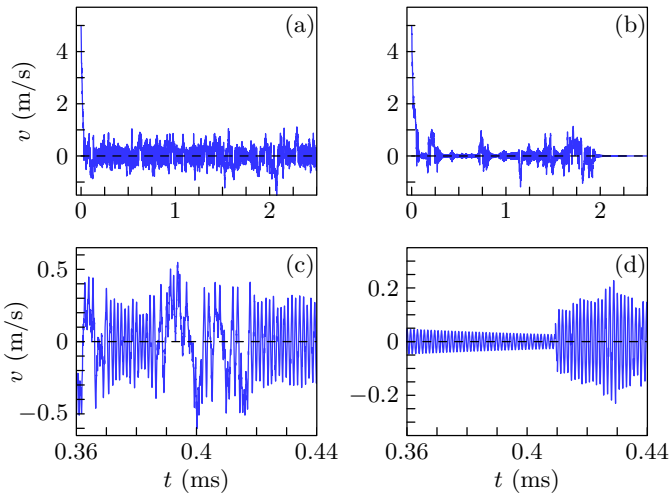


**Fig. 12.** Examples of time dependencies of velocity of sodium atoms used in the calculations of Figures 11a and 11b. Frames (c) and (d) show parts of these dependencies on a larger scale. Parameters: $\Omega_m = 2\pi \times 50$ MHz, $\beta = 0.5$, $\Omega_R = 2\pi \times 20$ MHz, $\delta = -2\pi \times 32$ MHz (a) and $\delta = -2\pi \times 38$ MHz (b), the initial velocity of atoms is $v_0 = 5$ m s$^{-1}$.

changes from time to time due to diffusion in momentum space. After all, the velocity becomes less than $v_r$. The period of oscillation of atoms near $z = 0$ is about 1.2 $\mu$s. They occur in the vicinity of field nodes. It is noteworthy that between sudden changes of the velocity due to spontaneous emission events, the movement of the atom looks like a damping oscillation in part (d) of Figure 12. In our opinion, this corresponds to a cooling of atoms without the participation of the spontaneous emission, predicted in [28], demonstrated experimentally in [29,30] and numerically simulated in [30] within the quasiclassical approach.

## 8 Conclusions

We theoretically studied the atomic motion in the field of counter-propagating frequency-modulated waves, one of which repeats the other. In this study, we analyzed the statistical properties of the atomic ensemble. We simulate the trajectory of each atom using the quasiclassical approximation. The stochasticity of the atomic motion arises due to spontaneous emission of photons by excited atoms. We calculate the atomic state by the Monte Carlo wavefunction method [31] and describe the mechanical motion of atoms by the Newton's law. As it turns out, accounting for the momentum diffusion of atoms, which is more pronounced for light atoms, can drastically change the overall pattern of the atomic motion in comparison with an analysis based only on the light pressure force exerted on atoms in the light waves. For example, sometimes we expect splitting of the atomic beam by orthogonally propagating FM waves instead of its collimation, if the derivative of the light pressure force with respect to velocity at zero velocity is positive.

Counter-propagating FM waves can cool atoms just as it occurs in a monochromatic standing wave but with a much more complicated dependence of the temperature on the carrier frequency detuning from the transition frequency of the atom. There is a region of parameters of the atom-field interaction where our calculations predict sub-Doppler cooling of the atoms. We explain this phenomenon by laser cooling without the participation of spontaneous radiation, as predicted in [28] and experimentally confirmed in [29,30] for helium atoms in the field of bichromatic waves.

The counter-propagating FM wave can form a one-dimensional trap for atoms provided that the intensity of these waves is large enough. The center of the trap is situated in the point where the phases of the waves are the same. At least two types of the atomic motion in the trap are possible. The motion of the first type is approximately characterized by the Maxwell distribution. The temperature of atoms is approximately an order of magnitude higher than the Doppler cooling limit and the size of the area where the atoms are confined is about 1 mm. Motion of the second type is characterized by the velocity distribution of atoms with two maxima and a small number of them around the zero velocity. This distribution cannot be characterized by a certain temperature. In this case the atoms move near the center of the trap in the area with the size of about 1 cm.

We modelled the behaviour of Na atoms for the cycling transition between two states [7]. Splitting of an atomic beam is based on zero light pressure force for nonzero velocity which obviously appears in any model of the atom interaction with FM field, and we expect this phenomenon to be observed for any multilevel model of the atom-field interaction. Our previous investigation of the trap based on counter-propagating light pulses [15] or stochastic waves [37], one of which repeats the other, led us to an hypothesis that any polychromatic counter-propagating waves with discrete spectrum or waves described by a stationary stochastic process, one of which repeats the other, can form a trap for atoms [36]. So, we expect that counter-propagating FM waves can also form a trap for atoms in the case of a multilevel model of the atom-field interaction too.

The interaction of an atom with the frequency modulated light waves may cause the rapid adiabatic passage between the atomic states if the conditions $\Omega_R/\Omega_m \ll \beta \ll (\Omega_R/\Omega_m)^2$, $\Omega_m \gg \gamma$ are satisfied. In this case, the FM waves acts on the atom like the sequences of the counter-propagating chirped pulses [34] each of them causes the transitions between the ground and excited states with the accompanying change of the momentum of the atom due to absorption or stimulated emission of a photon [38]. Detailed investigation of this regime of the atom-field interaction can be carried out when the description of the momentum diffusion process in the counter-propagating waves of very high intensity within the scope of the Monte Carlo wave function method will be developed. In this case, we expect the result of the atom-field interaction to be close to the result of interaction of an atom with the counter-propagating $\pi$-pulses [8,9,15].

The publication is based on the research supported by the goal-oriented complex program of fundamental researches of the National Academy of Sciences of Ukraine "Fundamental issues in creation of new nanomaterials and nanotechnologies" (Grant No. 3/17-H).

## Author contribution statement

All authors contributed equally to the paper.

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