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Queueing Models for Hospital Waiting Lists

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Results of some recent research into queueing models are applied to the hospital waiting-list problem to give some important insights into the likely implications of attempts to reduce waiting lists.

Key words: health services, queues

INTRODUCTION

Waiting lists have been an unsatisfactory feature of the National Health Service (NHS) since its inception in 1948. In part, they are a necessary evil in that they provide a pool of patients to ensure that expensive health-service resources do not lie idle for the want of suitable patients. However, waiting lists are usually much bigger than is necessary for this purpose.

Over the years, waiting lists have been the subject of much medical and political argument. For instance, in commenting on a recent survey by the British Medical Association, the secretary, Dr John Havard,[1] said, "The BMA survey shows a real increase in the length of time people with ordinary illnesses have to wait before they gain access to treatment". In reply, the Minister of Health[2] said, "All figures available to me show the number of patients being treated is going up and the number on waiting lists is going down.... Waiting lists have been an upsetting feature of the Health Service since 1948 and the Government has tried to tackle this problem since it came to power".

Waiting lists have also been the subject of a great deal of research. A review of much of this work, including over 100 references, is provided by Hicks.[3] One of the successes of this work has been the introduction of 'booked admission' systems in some hospitals (see, for example, Hindle[4] and Cox[5]). In these systems, patients are given appointments for admission to hospital when they are first put on the waiting list. Although the waiting list still exists, the booking of admissions leads to a more active management of the waiting list, and is more satisfactory from the patient's point of view. However, schemes of this sort are still in the minority, and many NHS hospitals still have a waiting-list problem.

One of the major problems in waiting-list management is identified by Culyer.[6] In describing rationing mechanisms in the NHS, he notes that "supply increases..." (which reduce waiting lists or waiting times) "...encourage general practitioners to refer more patients to hospital, and hospital doctors to assign more people to the waiting list, until a more or less 'conventional' waiting time is again reached". We shall refer to this feature of waiting lists as *feedback*.

In this paper, we use some recent research on queueing models to investigate some of the management implications of feedback in waiting lists. In particular, we examine some of the likely implications of a recent report of The College of Health,[7] in which patients and their general practitioners are encouraged to 'shop around' for short waiting lists. In order to facilitate this process, the report provides hospital waiting-list indicators (HWIs), namely tables of long and short waiting lists, by speciality and by health district.

THE QUEUEING MODEL

Modifying Kendall's classification of queueing systems slightly, the queueing model used in this paper is denoted by $M(\lambda_q)/G/S$. In this model, arrivals occur at random at a rate λ_q when there are q customers in the queue (this excludes customers in service), and λ_q is a linearly decreasing function of q,

$$\text{viz. } \lambda_q = \begin{cases} (N-q)\alpha & 0 \leqslant q \leqslant N \\ 0 & q > N \end{cases}.$$

Service times are sampled independently from any fixed probability distribution, and there are S servers.

This model is very similar to the traditional machine-interference problem. However, the results quoted here are much simpler than the nearest equivalent machine-interference results. They also allow any distribution of service time, whereas machine-interference results require the distribution to be of 'phase-type' (see, for example, Stewart and Marie[8]).

No exact results exist for the system $M(\lambda_q)/G/S$. However, research has shown that the steady-state behaviour of the number of customers in the queue can be approximated quite satisfactorily for most purposes by a Normal distribution with mean $E(q)$ and variance $V(q)$, as long as $E(q) - 2.6\sqrt{V(q)} > 0$. This condition ensures that the queue is hardly ever empty. Furthermore, approximate expressions for $E(q)$ and $V(q)$ depend only on N, S, α and the mean $E(t)$ and coefficient of variation $CV(t)$ of service time, namely:

$$E(q) \simeq N - \frac{S}{\alpha E(t)} \tag{1}$$

$$V(q) \simeq \frac{S}{2\alpha E(t)}(1 + CV(t)^2). \tag{2}$$

Expression (2) was derived from empirical results for $CV(t)^2 \leqslant 3$.

APPLICATION TO THE HOSPITAL WAITING-LIST PROBLEM

A consultant's waiting list can be considered as a queueing system as follows. The service is hospital inpatient treatment. Beds can usually be considered as servers as they usually provide the limit on number of people in service. The service time is essentially the patient's length of stay in the hospital bed, although a short turnover interval is needed to prepare the bed for the next patient.

Arrivals occur either as referrals from a general practitioner to the consultant's outpatient clinic and then to the waiting list if the consultant decides the patient needs inpatient care; or as an emergency case. As noted earlier, an important aspect of this arrival process is feedback, at both the general-practitioner referral stage and the outpatient-clinic stage.

To model this arrival process, suppose the population nominally served by the consultant is P, and the incidence rate (per person per unit time) of conditions suitable for inpatient treatment by the consultant is I. The occurrence of potential patients would then be expected to be at random, at a rate PI. We now consider two different models for the feedback mechanism.

In model 1, we assume that for each additional patient on the waiting list, a small proportion p of potential patients are discouraged from joining the waiting list. Thus, if there are q people on the waiting list, the arrival rate will be $PI(1 - qp)$.

In model 2, we use the concept of anticipated time on the waiting list, defined as the expected time required to serve the whole of the current waiting list. If the waiting-list size is q, this is simply $qE(t)/S$. We then assume that for each additional unit of anticipated time on the waiting list, a small proportion r of potential patients are discouraged from joining the waiting list. Thus, if there are q people on the waiting list, the arrival rate will be

$$PI\left(1 - \frac{qE(t)}{S}r\right).$$

In both models 1 and 2, arrivals will continue to be at random, whatever the waiting-list size. We are thus able to model waiting lists with feedback using the queueing model $M(\lambda_q)/G/S$, where $N = PI/p$, $\alpha = p$ for model 1, and $N = PIS/E(t)r$, $\alpha = E(t)r/S$ for model 2.

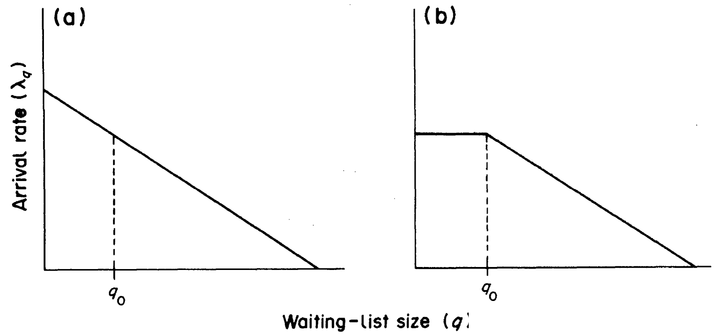

FIG. 1. *Arrival-rate functions*

In fact, the arrival-rate function does not have to be precisely linear throughout the whole range of possible q values. All that our results require is that λ_q is linear for the range of q values that usually occur. Hence an arrival-rate function of the form shown in Figure 1(b), where feedback does not start to operate until the number on the waiting list is q_0, can be modelled using the arrival-rate function shown in Figure 1(a), as long as the waiting-list size is rarely less than q_0.

MODEL VALIDATION

In this section, the $M(\lambda_q)/G/S$ model is considered for some real waiting lists. This discussion is based on conversations with doctors and administrators, and data collected at the Royal Berkshire Hospital in Reading. We shall consider in turn each of the three main assumptions of the $M(\lambda_q)/G/S$ model.

(*i*) *The number of servers*

The model assumes that beds are the main constraint on throughput, and hence can be considered as the servers. In a hospital where a different resource, say theatre time, provided the major constraint a different formulation would be required.

The number of beds assigned to a consultant usually remains fixed. However, there may be occasional temporary changes due to ward closures or bed-borrowing, and sometimes more permanent planned changes in bed allocations.

(*ii*) *Service-time distribution*

The model assumes that service times are sampled independently from a fixed probability distribution. As long as the types of patients and medical practice do not change, this will be a reasonable first assumption. In fact, deviations from this assumption can occur. For instance, Duncan and Curnow[9] have shown statistical evidence that, on some wards, the lengths of stay of patients decrease as the number of beds in use on the ward increases. Also, because doctors often select patients to give balanced operating theatre sessions, there can sometimes be correlations between service times.

(*iii*) *Arrival process*

The model assumes that arrivals occur at random at a rate that decreases linearly with waiting-list size. Data was collected over an 18-month period on nine waiting lists in order to investigate this assumption. The data is summarized in Table 1.

If the model is true, the number of patients joining a waiting list during a month should follow a Poisson distribution whose mean depends on the average number on the waiting list during the month. As the latter information was not available, and as patients were at most added to the waiting list once a week, the number on the waiting list at the start of the month was used as a substitute.

TABLE 1. *Monthly numbers on waiting lists and joining waiting lists*

	Consultant 1						Consultant 2						Consultant 3					
	Male		Female		Children		Male		Female		Children		Male		Female		Children	
Month	On WL	Join WL	On WL	Join WL	On WL	Join WL	On WL	Join WL	On WL	Join WL	On WL	Join WL	On WL	Join WL	On WL	Join WL	On WL	Join WL
1	133	25	97	12	63	20	104	19	72	11	85	27	30	12	22	6	86	47
2	139	14	99	4	62	9	109	21	75	14	84	20	25	15	23	10	77	35
3	143	32	87	7	55	11	123	35	78	24	75	41	30	12	25	5	81	27
4	157	26	82	15	49	11	141	22	97	16	83	55	24	10	24	9	60	34
5	170	24	82	8	43	24	144	27	96	22	100	37	23	14	16	8	44	40
6	175	16	78	13	56	23	158	21	102	21	99	37	23	12	15	16	53	43
7	177	22	79	8	64	20	164	28	107	12	89	22	28	11	25	5	78	46
8	188	18	76	15	60	18	165	24	107	12	75	25	27	14	19	7	92	72
9	188	13	78	9	62	29	169	18	106	16	68	38	31	15	13	6	124	50
10	178	21	76	12	77	17	168	12	107	17	70	27	29	6	10	3	134	26
11	193	14	81	15	76	28	161	21	107	13	62	49	25	10	13	12	120	42
12	195	22	88	9	79	24	159	23	112	19	82	28	26	16	15	12	122	44
13	200	25	86	11	77	16	166	25	119	26	83	36	30	11	17	5	129	37
14	215	18	87	22	65	20	169	20	123	13	83	30	23	6	12	9	114	26
15	199	11	90	14	62	11	168	14	120	8	77	22	21	10	10	10	100	25
16	184	23	87	19	56	17	165	19	112	12	71	29	19	10	17	6	70	13
17	179	12	92	7	57	16	160	14	106	10	66	33	13	7	11	3	41	19
18	184	22	99	16	73	20	162	11	111	13	96	24	17	11	13	6	59	21

TABLE 2. *Summary of GENSTAT analysis for nine waiting lists*

	Poisson model accepted?	Estimate of constant	Estimate of gradient	95% CI for gradient
Constant 1				
Male	Yes	38.9	−0.103	—
Female	Yes	18.2	−0.073	(−0.308, 0.162)
Children	Yes	10.7	+0.124	(−0.095, 0.343)
Constant 2				
Male	Yes	38.2	−0.109	—
Female	Yes	20.2	−0.046	(−0.186, 0.094)
Children	No	—	—	—
Constant 3				
Male	Yes	5.7	+0.224	(−0.111, 0.559)
Female	Yes	8.5	−0.052	(−0.386, 0.160)
Children	No	—	—	—

GENSTAT was used to analyse the relationship between numbers joining and numbers on the waiting list by fitting appropriate linear models. The results are summarized in Table 2. In two out of the nine lists, the Poisson distribution was rejected at a 5% significance level. In both these cases, the variability in numbers joining was much greater than could be explained by a Poisson distribution. For each of the remaining seven lists, the constant and gradient of the linear relationship were estimated.

For two of these seven lists, consultants 1 and 2 males, the statistical evidence indicated that the arrival rate decreased as the waiting-list size increased, i.e. the null hypothesis of no decrease was rejected at a 5% significance level (using a one-tailed test). However, for the remaining five lists there was insufficient evidence to reject the null hypothesis. One possible reason for this failure to reject is the power of the statistical test. In particular, if the 18 observations of waiting-list size do not vary much, there is little chance of detecting a change in joining rate. In fact, the two waiting lists that gave significant results were those with the largest variances of waiting-list size. For the remaining five lists, the problem of obtaining significant results is highlighted in Table 2 by the size of the 95% confidence intervals for the gradients.

Thus, what statistical evidence there is suggests that, for some real waiting lists, the observed process is adequately described by the model. However, given the limitations of the data, other possibilities also exist, e.g. feedback in which λ_q is non-linear, or in which the response of arrival rate lags behind changes in waiting-list size.

RESULTS

The formulae used here are deduced directly from the Normal approximation introduced earlier, (see Appendix for details). The results are in terms of three important aspects of waiting-list behaviour: the distributions of waiting-list size (q), anticipated time on waiting list (w) and acceptance rate (λ_q). Each distribution is summarized in terms of its mean and the range which contains roughly the central 95% of the distribution.

(i) Waiting-list size (q)

$$E(q) = \begin{cases} \dfrac{1}{p}\left[1 - \dfrac{S}{PIE(t)}\right] & \text{Model 1} \\[2ex] \dfrac{S}{rE(t)}\left[1 - \dfrac{S}{PIE(t)}\right] & \text{Model 2} \end{cases} \tag{3}$$

$$95\%\ \text{Range}\ (q) \simeq \begin{cases} E(q) \pm 2\left[\dfrac{S(1 + CV(t)^2)}{2PIpE(t)}\right]^{1/2} & \text{Model 1} \\[2ex] E(q) \pm 2\left[\dfrac{S^2(1 + CV(t)^2)}{2PIrE(t)^2}\right]^{1/2} & \text{Model 2.} \end{cases} \tag{4}$$

(ii) Anticipated time on waiting list (w)

$$E(w) \simeq \frac{E(q)E(t)}{S} \quad \text{Models 1 and 2} \tag{5}$$

$$95\% \text{ Range } (w) \simeq \frac{E(t)}{S} \cdot (95\% \text{ Range } (q)) \quad \text{Models 1 and 2.} \tag{6}$$

(iii) Acceptance rate (λ_q)

$$E(\lambda_q) \simeq \frac{S}{E(t)} \quad \text{Models 1 and 2} \tag{7}$$

$$95\% \text{ Range } (\lambda_q) \simeq \begin{cases} PI - PIp\ (95\% \text{ Range } (q)) & \text{Model 1} \\ PI - \dfrac{PIE(t)r}{S}\ (95\% \text{ Range } (q)) & \text{Model 2.} \end{cases} \tag{8}$$

Our main results are presented using examples that demonstrate the likely effects of certain management actions. All these results are summarized in Table 3. In the first example, the parameter values are chosen to correspond roughly to those of the male waiting lists of consultants 1 and 2.

Example 1—the present system

Service time has a mean of 1 week and a (coefficient of variation)^2 of 0.5. There are six beds available. The incidence rate (PI) of potential patients is 10 per week; the proportional reduction in arrival rate is $p = 0.0025$ for every extra person on the waiting list, or $r = 0.015$ for every extra anticipated week on the waiting list. The performance of this system is summarized in row (i) of Table 3, using the six performance measures introduced earlier. The last of these, 95% range (λ_q), is expressed as a percentage of $E(\lambda_q)$.

Thus both models predict a mean waiting-list size of 160, and actual size between 133.2 and 186.8 for 95% of the time. Similarly, anticipated time on the waiting list has a mean of 26.7 weeks, and is between 22.2 and 31.1 weeks for 95% of the time. Finally, the average acceptance rate of patients is 6.0 per week, and for 95% of the time the actual rate is within 11% of this. However, this does indicate that, for 5% of the time, the acceptance rate differs by at least 11% from the mean, which makes the state of the waiting list at the time of consultation of some importance in determining whether or not the patient will be placed on the waiting list.

Example 2—a waiting list without feedback

One of the important effects of feedback is that it helps to control waiting-list size. This example demonstrates how a waiting list would be expected to behave without feedback.

First, comparing with example 1, we note that with potential arrivals at a rate of 10 per week, no feedback, and an average service rate of six per week, the waiting list would simply keep

TABLE 3. *Waiting-list behaviour under various management actions*

		WL size		Anticipated time on WL (weeks)		Acceptance rate (per week)	
System	Model	$E(q)$	95% range (q)	$E(w)$	95% range (w)	$E(\lambda_q)$	95% range (λ_q)
(i) Present	1 and 2	160.0	(133.2, 186.8)	26.7	(22.2, 31.1)	6.0	(89%, 111%)
(ii) No feedback		160.0	(0.0, 560.0)	26.7	(0.0, 93.3)	6.0	(100%, 100%)
(iii) 2 extra beds	1	80.0	(49.0, 111.0)	10.0	(6.1, 13.9)	8.0	(90%. 110%)
(iv) 2 extra beds	2	106.7	(70.9, 142.4)	13.3	(8.9, 17.8)	8.0	(92%, 108%)
(v) 10% cut in $E(t)$	1	133.3	(104.0, 162.7)	20.0	(15,6, 24.4)	6.7	(89%, 111%)
(vi) 10% cut in $E(t)$	2	148.1	(117.2, 179.1)	22.2	(17.6, 26.9)	6.7	(90%, 110%)
(vii) Combining equal lists	1	160.0	(133.2, 186.8)	13.3	(11.1, 15.6)	12.0	(89%, 111%)
(viii) Combining equal lists	2	320.0	(282.1, 357.9)	26.7	(23.5, 29.8)	12.0	(92%, 108%)
(ix) Combining unequal lists	1	120.0	(91.0, 149.0)	8.6	(6.5, 10.6)	14.0	(90%, 110%)
(x) Combining unequal lists	2	280.0	(235.7, 324.3)	20.0	(16.8, 23.2)	14.0	(93%, 107%)
(xi) Earlier feedback	1	80.0	(53.2, 106.8)	13.3	(8.9, 17.8)	6.0	(89%, 111%)
(xii) Earlier feedback	2	80.0	(53.2, 106.8)	13.3	(8.9, 17.8)	6.0	(89%, 111%)
(xiii) (i) −16.2%	1 and 2	113.5	(84.2, 142.8)	18.9	(14.0, 23.8)	6.0	(90%, 110%)
(xiv) (xi) +16.2%	1 and 2	113.5	(88.6, 138.4)	18.9	(14.8, 23.1)	6.0	(88%, 112%)

growing. We must therefore assume that a constant proportion of patients are deterred, so that the average arrival rate is less than six per week. It is then reasonable to model this system as $M/E_2/6$, where E_2 is chosen because it has (coefficient variation)^2 of 0.5.

Extrapolating from the tables of Hillier and Yu,[10] the system would need to have a traffic intensity of about 0.995 to have a mean waiting-list size of 160, as in example 1. The performance parameters shown in row (ii) are then derived from these tables. Although this system is desirable in that it no longer has varying standards for joining the waiting list, unfortunately it also has high variability of both waiting-list size and anticipated time on the waiting list.

Example 3—increasing the number of beds

One traditional response to long waiting lists has been to increase bed provision in the expectation that this will clear the backlog. For instance, if two extra beds are provided in example 1, then in theory, in 80 weeks, they should be able to treat 160 patients, and hence reduce the waiting list to zero. In practice this does not happen because of the reaction of the arrival rate to shorter waiting lists. Rows (iii) and (iv) show the predicted effects of two extra beds for the two feedback models. In both cases, the additional beds improve the service, but substantial waiting lists and waiting times still remain.

Example 4—decreasing mean service-time

Decreasing mean service-time is also a possible method for eradicating waiting lists. Rows (v) and (vi) show that both models predict that a 10% decrease in mean service-time would increase mean acceptance-rate by 11% but would have very little effect on waiting-list size and waiting time.

Example 5—combining two equal lists

There are some benefits from combining two equal waiting lists, although the two models produce very different results. For example, if each of the lists is as in example 1, then model 1 predicts that combined waiting-list size and anticipated waiting time would be halved—see row (vii). Model 2, row (viii), on the other hand, shows benefits only as reduced variabilities in acceptance rate and anticipated waiting time. In the context of a significant reorganization of this sort, model 2 is the more likely.

Example 6—combining two unequally resourced lists

This example highlights some of the likely implications of the College of Heath[7] proposals that general practitioners and their patients should 'shop around' for short waiting lists. Row (ix) shows the effect of combining the lists in examples 1 and 3 [see rows (i) and (iii)] using model 1. Row (x) combines the same two lists using model 2; this time compare with rows (i) and (iv). Concentrating on model 2, which is more likely in this context, we see that a simple averaging of waiting times is obtained. The combined acceptance-rate does not change, although as the two identical catchment populations now share the same resources, they have equal acceptance-rates.

Examples so far have used arrival-rate functions of the form shown in Figure 1(a). As noted earlier, our results can also be used for arrival-rate functions of the form shown in Figure 1(b). For instance, row (i) also predicts the behaviour of a system with an arrival rate that stays constant at 7.5 per week until the waiting-list length is 100, and then falls linearly according to model 1, with $p = 0.0025$. The model 2 equivalent is that arrival rate is constant at 7.5 per week until anticipated waiting time is 16.7 weeks, and it then falls linearly with $r = 0.015$.

Arrival rates of the form in Figure 1(b) correspond to situations in which general practitioners and consultants start to consider alternatives to inpatient care when waiting lists reach q_0.

Example 7—introducing feedback earlier

Rows (xi) and (xii) predict the effects of an earlier reaction by general practitioners and/or consultants to growing waiting lists. Here the arrival rate starts to be reduced when the waiting-list length is 20, or the anticipated waiting time is 3.3 weeks. The values of p and r used are unchanged from example 1. Comparing with row (i), we can see the same dramatic improvement in predicted waiting-list behaviour for both models, at no cost to patients in terms of acceptance rates.

Example 8—combining two differently managed lists

This final example considers further some of the College of Health[7] proposals. Here we consider one area which has a waiting list in which feedback was introduced late, as in example 1, and a second area which has a waiting list in which feedback was introduced earlier, as in example 7. The above proposal will cause some of the workload from the first area to go to the second. If the two areas do not change their waiting-list management policies, the two waiting lists will eventually settle to equal lengths and equal anticipated waiting times, by which time the second area will be doing 16.2% of the first area's work. The behaviour of the two new waiting lists is shown in rows (xiii) and (xiv). Although the two waiting lists are now almost identical in their behaviour, the second area has suffered a substantial reduction in service. Its anticipated waiting time has increased by 6.6 weeks, and more importantly, the acceptance rate from its own catchment population has dropped by 16.2%.

CONCLUSIONS

We have estimated fairly precisely the implications of certain management actions for waiting lists in which arrival rates decrease linearly as waiting-list size increases. Although some of the precision of our results will be lost for waiting lists that have other sorts of feedback, we nevertheless believe that a number of important conclusions can be drawn from the results and examples presented here.

Feedback is an important factor in waiting-list management. Feedback makes it possible to maintain a pool of patients for admission and at the same time avoid very long and unstable waiting lists. However, it also means that traditional 'solutions' to long waiting lists rarely 'solve' the problem (although they do improve throughput). A much more effective method for controlling waiting lists would be to 'tighten up' the feedback mechanisms that already exist.

The College of Health proposals do not seem to be particularly well advised. On occasions, when their effect is to combine two unequally resourced waiting lists, there will be some benefits. However, when their effect is to combine two differently managed waiting lists, the outcome at best will be unfair, and they will probably undermine what good management practices already exist.

APPENDIX

The queueing model $M(\lambda_q)/G/S$ has arrival-rate function

$$\lambda_q = (N - q)\alpha \quad \text{for} \quad 0 \leqslant q \leqslant N.$$

Comparing this with the two formulations of the waiting-list problem gives

$$\text{Model 1: } N = 1/p, \quad \alpha = PIp;$$

$$\text{Model 2: } N = \frac{S}{rE(t)}, \quad \alpha = \frac{PIE(t)r}{S}.$$

Substituting these values into the expressions (1) and (2) gives the following results.

Waiting-list size (q)

$$E(q) \simeq \begin{cases} \dfrac{1}{p} - \dfrac{S}{PIpE(t)} & \text{Model 1} \\[2ex] \dfrac{S}{rE(t)} - \dfrac{S^2}{PIE(t)^2 r} & \text{Model 2} \end{cases}$$

which gives expression (3) after minor rearrangement.

$$V(q) \simeq \begin{cases} \dfrac{S(1 + CV(t)^2)}{2PIpE(t)} & \text{Model 1} \\[2ex] \dfrac{S^2(1 + CV(t)^2)}{2PIrE(t)^2} & \text{Model 2.} \end{cases}$$

Now the central 95% of a Normal distribution is contained in the range $E(q) \pm 2\sqrt{V(q)}$, so substituting in the above expression for $V(q)$ and minor rearrangement gives expression (4).

Anticipated time on waiting list (w)

We define this to be the time that a patient would spend on the waiting list if the average service time of patients in front of him is equal to the average service time of all patients, and if patients are treated in the order of joining the waiting list. The residual service times of patients already in service, which will be relatively small, are ignored. Thus the anticipated waiting time (w) of a customer who finds a waiting-list size of q is given by

$$w = q\frac{E(t)}{S}.$$

In order to calculate the precise distribution and statistics of w, the different values of w should be weighted by the relative numbers of patients who have that value of w, which are given by the λ_qs. However, quite a high degree of accuracy is achieved, and the expressions are much simpler if we use

$E(w) \simeq$ anticipated waiting time if waiting-list size is $E(q)$,
95% range $(w) \simeq$ range of anticipated waiting times associated with values of waiting-list size in its 95% range.

Expressions (5) and (6) then follow immediately.

Acceptance rate (λ_q)

The acceptance rate is defined to be the rate at which patients are put onto the waiting list, so that

$$\text{acceptance rate} = \lambda_q \text{ when } q \text{ on waiting list.}$$

To obtain $E(\lambda_q)$, note that at steady-state,

$$\text{mean acceptance-rate} = \text{mean service-rate.}$$

As the waiting list is almost never empty,

$$\text{mean service-rate} \simeq \frac{S}{E(t)}.$$

By definition, the 95% range (λ_q) is the range of arrival rates associated with the 95% range (q). But for any q, the associated arrival rate is

$$\lambda_q = (N - q)\alpha,$$

and substituting in the appropriate values of N and α gives expression (8).

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