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Editor

Multiple Criteria Decision Making

Techniques, Analysis and Applications

Springer

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Anand would like to dedicate this book to the memories of his good friend Moumen Darcherif, former Vice President of the Paris-Seine University, France. The book was conceptualized with Moumen; however, sadly lost him during the journey.

Preface

Multiple criteria decision-making (MCDM) provides a mathematical methodology that incorporates the values of decision-makers and stakeholders along with the technical information to select the best solution for a given problem. In general, decision-making occurs in all domains; however, it is challenging to deal with different tangible and intangible alternatives from diverse aspects with an intention of selecting the best one amongst them. The MCDM techniques are indispensable for decision-makers as the evaluation of a problem with different alternatives has conflicting and incommensurable objectives and is required to be optimized simultaneously. Importantly, the use of these techniques reduces subjectivity because of the psychology–human interaction. The aim of MCDM is to obtain the optimal alternative/choice that has the highest degree of satisfaction for most of the criteria. The growing recognition of MCDM approaches has motivated several researchers to further develop, test and apply them in various fields. In recent times, digital automation, machine learning, big data, IoT and artificial-intelligence-based methods offer promising solutions to the growing complexity. Today's strategic decision-making needs to be re-evaluated and addressed through advanced MCDM and integrated approaches of AI, big data and IoT, to provide more realistic and robust solutions to the current problems.

The edited book intends to provide a platform for interdisciplinary state-of-the-art discussion on MCDM with a focus on critical literature, underlying principles of methods and models, solution approaches, testing and validation, real-world applications, case studies etc. The book may provide guidelines to the potential MCDM researchers about the choice of approaches for dealing with the complexities and modalities. The contributions of the book may further help to explore new avenues leading towards multi-disciplinary research discussions.

Every chapter submitted to the book has been critically evaluated by at least two expert reviewers. The critical suggestions by the reviewers helped and influenced the authors of the individual chapter to enrich the quality in terms of experimentation, performance evaluation, representation etc. The book may serve as a valuable reference for the researchers working in the domain of MCDM.

Chapter "MIVES: A Multi-Attribute Value Function-Based Methodology for Sustainability Assessment" by Biswal et al. highlights that any process or product design depends on the triple bottom approach of economic, environment and social criteria; however, sustainability assessment remains an important aspect of it. It further emphasizes the necessity of handling the associated subjectivity and difficulty in assessment. Accordingly, the chapter discusses MIVES (Modelo Integrado de Valor para una Evaluación Sostenible), a multi-attribute value function-based methodology framework for the necessary sustainability assessments. The applications of this methodology in construction, aviation, education and biomass processing industries are thoroughly discussed, which further underscores the need for a framework for sustainability evaluation in different domains of various industries.

One of the very recent MCDM methods referred to as the base-criterion method (BCM) is thoroughly discussed by Haseli and Sheikh in chapter "Base Criterion Method (BCM)". The discussion includes the detailed theoretical and mathematical formulation of the BCM. The essential characteristic of this method is that it removes a large number of unnecessary comparisons by dividing pairwise comparisons into two categories, viz., base comparisons and final comparisons. Importantly, the chapter provides illustrative examples showcasing the problem-solving process using the BCM method. The examples involving several criteria are associated with staff selection, product development and mode of transportation of products to the market. The examples are utilized to highlight the characteristics of the method; however, the chapter distinctively provides the limitations of the method along with recommendations.

DEX (Decision EXpert) is a method combining multi-criteria decision analysis and artificial intelligence suited particularly for sorting/classification decision problems. It is characterized by its hierarchical, qualitative, rule-based, multi-criteria decision modeling approaches. Chapter "DEX (Decision EXpert): A Qualitative Hierarchical Multi-criteria Method" by Marko Bohanec described DEX from the theoretical and practical viewpoint, and further explained it in terms of motivation, history, software, applications and method extensions. The work is supported and illustrated using three examples: a didactic example of employee selection and two real-world industrial applications of choosing a raw-material location and assessing electric energy production technologies, respectively. Importantly, the chapter in detail lists the limitations and the associated solutions as well.

The MCDM methods, such as the analytic hierarchy process (AHP) and the technique for order of preference by similarity to ideal solution (TOPSIS), work on the crisp data. The data may be inadequate and vague when dealing with real-world problems. This issue is addressed in chapter "Analysis of Fuzzy AHP and Fuzzy TOPSIS Methods for the Prioritization of the Software Requirements" by Nazim et al. by successfully applying them under a fuzzy environment in different areas, like management science, software engineering etc. The chapter attempts to present a comparison of the accuracy of the fuzzy AHP and fuzzy TOPSIS methods using different cases of software requirements prioritization problems. The chapter describes detailed mathematical formulations of the fuzzy AHP and fuzzy TOPSIS methods. The authors have also thrown light on limitations as well as possible enhancement of the study in terms of the number and size of the test cases.

Digital image forensic science is one of the very niche areas working towards checking the authenticity of digital images. Even though several algorithms have been developed to verify the forged images, the evaluation and selection of the digital image forensic tools based on different features, like error-level analysis, metadata analysis etc., are still required to be explored. This issue is addressed by Parveen et al. in chapter "A Fuzzy-Based Multi-Criteria Decision-Making Approach for the Selection of Digital Image Forensic Tools". It discusses an algorithm specifically developed for the selection of the digital image forensic tools based on the ranking values. The ranking values of the digital image forensic tools are computed using TOPSIS method by using the triangular fuzzy numbers. The utilization of the proposed method is discussed with the help of an example in which the tools, such as FotoForensics, JPEGsnoop, Forensically, Ghiro and Izitru, are utilized during the analysis.

The decision matrix in the multi-criteria decision analysis (MCDA) problem necessarily involves a set of criteria and alternatives. They may have different units of measurement and are not suitable for a direct comparison. In chapter "Why Does the Choice of Normalization Technique Matter in Decision-Making", Shekhovtsov et al. underscore this issue and highlight that most of the MCDA methods require normalization of the decision matrix which can contribute to change in the final result. The chapter provides a significant investigation to show the fundamental differences between the five most common and prominent normalization techniques, viz., the minimum–maximum method, the maximum method, the sum method, the vector method and the logarithmic method. The methods have been statistically compared by using six randomly generated diverse data sets, differing in range, size and sign. The chapter highlights that the characteristics of data sets have a significant impact on normalization results. It further highlights the need for further investigation in interval normalization as well as fuzzy numbers.

Aggregation–disaggregation or ordinal regression approach is currently considered as an important tool at the disposal of potential analysts and decision-makers when addressing MCDM problems. Chapter "Bipolar Multicriteria Aggregation-Disaggregation Robustness Approach: Theory and Application on European e-Government Benchmarking" by Siskos et al. proposed a bipolar robustness control approach, implemented in conjunction with the UTASTAR method characterized by an additive value function referred to as the multi-criteria evaluation model. The methodology is applied to the problem of an e-government readiness evaluation in Europe, resulting in the ranking of 22 European countries. This application is based on one of the author's earlier developed multi-criteria e-government modeling approaches. The authors bring attention to certain robustness issues of the ordinal regression framework and the way that the preferential parameters are estimated through the UTA-type inference engine as well. The chapter in detail discusses the additive value model and UTASTAR method, and the principle of bipolar ordinal regression process and the robustness control.

Chapter "The COMET Method: Study Case of Swimming Training Progress" by Więckowski and Watróbski in detail describes an MCDA technique referred to as characteristic objects method (COMET). The theory of fuzzy numbers combined with the COMET method is used to create an evaluation model with complete knowledge and a certain degree of inherent uncertainty. The COMET and linear regression method have been used in a practical application in determining the trend of a swimmer's form. The use of the method is quite important and practically significant as several factors influence the preparation of the top form for the main swimming competition. The direct preparation period lasts for about three months, during which the competitor swims hundreds of kilometers on different intensities. Using the COMET, the values of attributes for each swimmer are introduced. The obtained model allows a broader analysis of the progress in terms of particular criteria sensitivity and robustness analysis.

Chapter "Brown-Gibson Model as a Multi-criteria Decision Analysis (MCDA) Method: Theoretical and Mathematical Formulations, Literature Review, and Applications" by Yimen et al. highlighted that different models of the MCDA method referred to as the Brown-Gibson model have so far been applied in various engineering and science fields, and different versions. The authors have provided a rich, complete and critical state-of-the-art literature survey of different versions. It includes theoretical and mathematical formulations of different versions, viz., original Brown-Gibson model, Buffa and Sarin version of Brown-Gibson model, the extended Brown-Gibson model, the Yimen & Dagbasi version of Brown-Gibson model, analytical hierarchy process (AHP)-integrated Brown-Gibson model and fuzzy Brown-Gibson model. An illustrative application of the original Brown-Gibson model in determining the optimal location to set a commercial center in Cameroon is provided with the choice associated with certain critical factors, objective factors and subjective factors. The authors highlighted that two facts confirm the importance of the Brown-Gibson model, viz., the model has seen several developments and applications since its inception and the model has a unique ability of combining the objective and subjective factors in decision-making.

In chapter "A Grey Approach for the Computation of Interactions Between Two Groups of Irrelevant Variables of Decision Matrices" Zakeri et al. emphasized that the existing MCDM methods merely provide solutions for the one-stage decision-making procedure and do not take other effective variables outside of the decision matrix into account, while in real-world processes, the decisions always have an impact on the variables where they appear to be irrelevant. The chapter aims at finding a mathematical solution to compute the impact between two irrelevant decision matrices in a complex decision-making problem using MCDM methods. The proposed strategies interaction model (SIM) approach is applied to a case of supplier selection and the strategies of the firm in which the interaction of selected strategies is investigated on the selection of the best supplier. The inherent uncertainty is handled using a four-section approach implemented as a grey framework and deals with grey Entropy, grey TOPSIS and the grey strategies interaction model.

Chapter "Statistical Analysis of KMM Program—An Educational Intervention" by Vaidya et al. highlighted that the educational interventions are intended to help struggling students by addressing their behavioral issues and social skills. Several criteria and factors need to be considered. In this regard, the chapter presents the complete life cycle of the intervention process implemented by the Keep Moving

Movement (KMM) as a pilot study. The impact of the KMM programme is analysed using correlation analysis, factor analysis and paired t-test. The group-wise and student-wise analysis of students reveals significant positive changes in positive thinking and willingness. The study provides possible extension towards a larger set of students to improve positive thinking, confidence and willingness.

Pune, India Anand J. Kulkarni

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Contents

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About the Editor

Anand J. Kulkarni holds a Ph.D. in Distributed Optimization from Nanyang Technological University, Singapore, MS in Artificial Intelligence from the University of Regina, Canada, Bachelor of Engineering from Shivaji University, India, and Diploma from the MSBTE, Mumbai. He worked as a Post Doctorate Research Fellow at Odette School of Business, University of Windsor, Canada. Dr. Kulkarni has worked with Symbiosis International University, Pune, India for over six years. Currently, he is a Professor & Associate Director at the Institute of AI at MITWPU. His research interests include optimization algorithms, multi-agent systems, complex systems, swarm optimization, and self-organizing systems. Anand pioneered socio-inspired optimization methodologies such as Cohort Intelligence, Ideology Algorithm, Expectation Algorithm, and Socio Evolution & Learning Optimization Algorithm. He is the founder and chairman of Optimization and Agent Technology Research Lab and has over 70 research papers in journals and conferences, 04 authored and 08 edited books to his credit. Dr. Kulkarni is the lead editor for the Springer and Taylor and Francis book series. He regularly writes on Artificial Intelligence in several newspapers and magazines. Dr. Kulkarni has delivered expert research talks in many countries such as the USA, Canada, Singapore, Malaysia, India, and France.

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Abbreviations

3D	Three-dimensional
AHP	Analytic Hierarchy Process
AIFD	Active Image Forgery Detection
AMT	Advanced Manufacturing Technology
ANP	Analytic Network Process
APM	Assessment Preference Measure
AQ	Algorithm Quasi-optimal, a machine rule learning algorithm
ARP	Average Range of preferential Parameters
ARRI	Average Range of the Ranking Index
AS/RS	Automatic Storage and Retrieval System
ASI	Average Stability Index
AVG	Average
BPE	Biomass Processing Enterprise
BREEAM	Building Research Establishment Environmental Assessment Method
C	Criteria
CC	Closeness Coefficients
CDPC	Consistency-Driven Pairwise Comparisons, an MCDM method
CFI	Critical Factor Index
CFM	Critical Factor Measure
COMET	Characteristic Objects Method
COPRAS	Complex Proportional Assessment
COs	Characteristic Objects
CSM	Computer Software Measure
CT	Conventional Technology
CTE	Cost and Time Effectiveness
DECMAK	DECision MAKing, an early predecessor of DEX
DEX	Decision EXpert, a qualitative MCDM method
DEXi	Software implementing the DEX method
DIF	Digital Image Forensic
DM	Decision Maker

DRSA	Dominance-based Rough Set Analysis, an MCDM approach
DSS	Decision Support System
EBG	Extended Brown-Gibson
EC	Effective Cost
EEA	European Environmental Agency
ELA	Error Level Analysis
ELECTRE	ÉLimination et Choix Traduisant la REalité, a family of MCDM methods
ENF	Effective Non-Financial
ERA	Extreme Ranking Analysis
ET	Effective Time
F	Fair
FDM	Fuzzy Decision Matrix
FLSI	Fuzzy Location Selection Index
FNIS	Fuzzy Negative Ideal Solution
FOFC	Fuzzy Objective Factor of Alternative
FOFM	Fuzzy Objective Factor Measure
FPIS	Fuzzy Positive Ideal Solution
FR	Functional Requirement
FSFM	Fuzzy Subjective Factor Measure
FST	Fuzzy Set Theory
FUSE	Fuzzy Synthetic Extent
G	Good
GDP	Gross Domestic Product
GRIHA	Green Rating for Integrated Habitat Assessment
G-TOPSIS	Grey-TOPSIS
GUV	the Grey Uncertainty Value
H	High
HINT	Hierarchical INduction Tool, a machine-learning method for developing DEX models from data
ICT	Information and Communications Technology
IEC	Ineffective Cost
IES	Institute Examination System
IET	Ineffective Time
INF	Ineffective Non-Financial
JPEG	Joint Photographic Expert Group
L	Low
LCA	Life cycle assessment
LEED	Leadership in Energy and Environmental Design
LM	Location Measure
LP	Linear Programming
LPM	Location Preference Measure
M	Medium
MA	Metadata

MACBETH	Measuring Attractiveness by a Categorical Based Evaluation Technique, an MCDM method
MADA	Multi-Attribute Decision Analysis
MCDA	Multi-Criteria Decision Analysis
MCDM	Multi-criteria Decision Making
MCHP	Multi-Criteria Hierarchy Process, a hierarchical MCDM approach
MG	Medium Good
MIVES	Modelo Integrado de Valor para una Evaluación Sostenible
mmol	Millimole
MODA	Multi-Objective Decision Analysis
MP	Medium Poor
MSPM	Manufacturing System Preference Measure
NFR	Non-functional Requirement
NIS	Negative Ideal Solution
NPV	Net Present Value
OFC	Objective Factor Cost
OFM	Objective Factor Measure
P	Poor
PIFD	Passive Image Forgery Detection
PIS	Positive Ideal Solution
PROMETHEE	Preference Ranking Organization Method for Enrichment of Evaluations
PSPM	Pumping System Preference Measure
QFD	Quality Function Deployment
QQ	Qualitative-Quantitative, an approach to ranking of alternatives using a DEX model
QSPM	Quantitative Strategic Planning Matrix
R&D	Research and Development
RARR	Ratio of the Average Range of the Ranking
RMS	Reconfigurable Manufacturing Systems
ROR	Robust Ordinal Regression
S	Strong
SDGs	Sustainability Development Goals
SFM	Subjective Factor Measure
SIM	Strategies interaction model
SO	Strengths Opportunities
SPRI	Statistical Preference Relations Index
SR	Software Requirement
ST	Strengths Threats
SWOT	Strengths, Weaknesses, Opportunities, Threats
T	Tools
TFN	Triangular Fuzzy Number
TFNs	Triangular Fuzzy Numbers
TOPSIS	The Technique for Order of Preference by Similarity to Ideal Solution

TS	Traditional System
TVM	Time Value of Money
UTA	UTilités Additives
VG	Very Good
VH	Very High
VIKOR	VlseKriterijumska Optimizacija I Kompromisno Resenje
VL	Very Low
VP	Very Poor
VS	Very Strong
VW	Very Weak
W	The subjective factor weight
WN	Weighted Normalized Fuzzy Decision Matrix
WO	Weaknesses Opportunities
WT	Weaknesses Threats

Symbols

X_{min}	Indicator value generating minimum satisfaction value
X_{max}	Indicator value associated to maximum satisfaction
X_{ind}	Value of indicator for which value function is to be calculated
$X_{n \times m}$	Decision matrix, where n stands for the number of alternatives and m for the number of criteria
x_{ij}	Element of the decision matrix, where $i = 1, 2, \ldots, n$ is an index of alternative and $j = 1, 2, \ldots, m$ is an index of criteria
r_{ij}	Element of the normalized decision matrix, where $i = 1, 2, \ldots, n$ is an index of alternative and $j = 1, 2, \ldots, m$ is an index of criteria
V	Value function
P_i	Shape factor that is decided on the shape of the curve
C_i	Axis value of the point where the curve changes its direction
K_i	Response value to the Ci
A	A constant, usually considered to be 0
B	A constant, limits the function to a range of 0 to 1
$\mathcal{A} = \{A_1, A_2, \ldots, A_q\}$	Alternatives
$A_i \in \mathcal{A}, A_i = \{a_{x,i} \in E_x, \forall x \in X\}$	An alternative
$a_{x,i} \in A_i$	Value of A_i assigned to attribute x
$B_x \subset D_x$	Subset of bad values of attribute x
$C_y = \prod_{x \in S(y)} D_x$	Domain of f_y
D_x	Value scale of attribute x

E_x	Range of values that can be assigned to attribute x
E_y	Range of f_y
$e = (\mathbf{x}, y) \in T_y$	Elementary decision rule, an entry in T_y
F	Set of aggregation functions of a DEX model
$\mathcal{F}_y$	Fuzzy distributions over D_y
$f_y : C_y \to E_y$	Aggregation function associated with attribute y
g_y	An approximation of f_y
$G_x \subset D_x$	Subset of good values of attribute x
$I_i \subset A_i$	Subset of values of A_i, assigned to input attributes
I_y	Set of intervals over D_y
$m_x = \|D_x\|$	Number of categories of scale D_x
$M = (X, D, S, F)$	A DEX model
$N_x \subset D_x$	Subset of neutral values of attribute x
$O_i \subset A_i$	Subset of values of A_i, assigned to output (aggregate) attributes
$\mathrm{ord}(v_{x,i}) = i$	Ordinal value of $v_{x,i}$
$P(x)$	Set of parents of attribute x
$\mathcal{P}_y$	Probability distributions over D_y
$r_y = \|C_y\|$	Size of C_y and the corresponding T_y
$S : X \to 2^x$	Descendant function
$S(x)$	Set of descendants of attribute x
$\mathcal{S}_y$	The power set of D_y
$T_y = \{(\mathbf{x}, y), \mathbf{x} \in C_y, y \in E_y\}$	Decision table associated with attribute y
$v_{x,i} \in D_x$	i-th qualitative value (category) of attribute x
$v_{x,i} \preccurlyeq v_{x,j}$	Weak preference relation
$w, w_i \in \mathcal{R}$	Relative weight (importance) of an attribute
$x, x_i, y \in X$	An attribute
$X = \{x_1, x_2, \ldots, x_n\}$	Set of attributes
$\omega \in [-0.5, +0.5]$	An offset to qualitative value v
$\sum$	Summation
$\otimes$	Fuzzy multiplication operator
min ( )	Returns minimum value
max ( )	Returns maximum value
α	The objective factor decision weight
$\mu_{\tilde{A}}$	is the value of the membership function
C_r	set of the fuzzy numbers, where C_i represents each criterion
CO	Characteristic Objects as Cartesian Product
a_{ij}	element of the matrix of expert judgement, where $i = 1, 2, \ldots, n$ is and index of row in matrix and $j = 1, 2, \ldots, m$ is an index of matrix' column

SJ_i	is the sum of the *i-th* row of matrix of expert judgement
y	is the value of linear function
β_0	is the coefficient of the independent variable in linear function
β_1	is a directional coefficient in linear function
x	is the independent variable in linear function
$\otimes G_1$	A grey number
$\left[\underline{G_1}, \overline{G_1}\right]$	Grey interval
$\underline{G_1}$	Grey lower bound
$\overline{G_1}$	Grey upper bound
e	Entropy
w	Weight
S^{max}	Positive ideal alternative
S^{min}	Negative ideal alternative
γ_{oi}	The grey relation coefficient
C_i	The grade of grey relation
$\otimes P = \left[\underline{P_{ij}}, \overline{P_{ij}}\right]$	A normalized grey number
$N_D = \left[N_{\underline{G}_{ij}}, N_{\overline{G}_{ij}}\right]$	The normalized decision matrix
D	The decision matrix
ℓ	The distance between the elements of each cloud with lower and upper bound
ϱ	GUV
ξ	The coefficient of uncertainty/probability