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Toshiyuki Kobayashi • Toshihisa Kubo
Michael Pevzner

Conformal Symmetry Breaking Operators for Differential forms on Spheres

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Toshiyuki Kobayashi
Kavli IPMU and
Graduate School of Mathematical Sciences
The University of Tokyo
3-8-1 Komaba, Meguro, Tokyo, Japan

Toshihisa Kubo
Ryukoku University
Kyoto, Japan

Michael Pevzner
Mathematics Laboratory, FR 3399 CNRS
University of Reims-Champagne-Ardenne
Reims, France

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Contents

Summary

We make a systematic study of all possible conformally covariant differential operators that transform differential forms on a Riemannian manifold X into those on a submanifold Y with focus on the model space $(X,Y) = (S^n, S^{n-1})$.

We accomplish the complete classification of all such conformally covariant differential operators, and find explicit formulæ for these new matrix-valued operators in the flat coordinates in terms of basic operators in differential geometry and classical hypergeometric polynomials. Resulting families of operators are natural generalizations of the Rankin–Cohen brackets for modular forms and Juhl's operators from conformal holography.

The matrix-valued factorization identities are also established for all possible combinations of these new conformally covariant differential operators with known operators for the $X = Y$ case such as the Yamabe, Paneitz–Fradkin–Tseytlin, GJMS, and Branson operators.

The main machinery of the proof is the "F-method" which has been recently introduced in [Contemp. Math., 2013] and [Differential Geom. Appl., 2014] by the first author and developed in Kobayashi–Ørsted–Somberg–Souček [Adv. Math., 2015] and Kobayashi–Pevzner [Selecta Math., 2016] in various settings. It is a general method to construct intertwining operators between C^∞-induced representations or to find singular vectors of generalized Verma modules in the context of branching laws, as solutions to differential equations given by the "algebraic Fourier transform of Verma modules". We extend the F-method to the matrix-valued case in the general setting, which could be applied to other problems as well.

A short summary of the main results was announced in [C. R. Acad. Sci. Paris, 2016].