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Abelian Group Theory

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PREFACE

A conference on Abelian Group Theory was held at the Manoa Campus of the University of Hawaii from December 28, 1982 to January 4, 1983. It was probably the best attended conference on Abelian Group Theory to date with 55 participants from all over the world and the busiest one with 49 talks. A special feature were general interest lectures by Hyman Bass, Columbia University, on "Non-linear Algebra", and by Claus Michael Ringel, Universität Bielefeld, on "Representations of Algebras". The Conference offered surveys by Laszlo Fuchs, Tulane University, on "Torsion Modules over Valuation Rings", Fred Richman, New Mexico State University, on "Mixed Groups", Paul Eklof, University of California at Irvine, on "Set Theory and Structure Theorems", Rüdiger Göbel, Universität Essen on "Endomorphism Rings", and Lee Lady, University of Hawaii, on "Torsion Free Groups of Finite Rank". The research reports attested to lively activity in the traditional and in new areas of inquiry within and around Abelian Group Theory. The best represented groups were those of finite rank without torsion, a field employing increasingly sophisticated tools from ring theory and algebraic number theory. The use of set theoretic methods continues to flourish after the pioneering work of Saharon Shelah in the 1970s, and is delivering impressive results.

This volume contains the papers of the participants of the Conference and a number of additional articles of others who could not or did not come. It reflects faithfully and almost completely the present status of Abelian Group Theory and should be invaluable to those in Abelian Groups and those who want to be informed.

Rüdiger Göbel, Universität Essen
Lee Lady and Adolf Mader, University of Hawaii

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The Editors

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