Editorial Policy

for the publication of monographs

In what follows all references to monographs, are applicable also to multiauthorship volumes such as seminar notes.

§ 1. Lecture Notes aim to report new developments - quickly, informally, and at a high level. Monograph manuscripts should be reasonably self-contained and rounded off. Thus they may, and often will, present not only results of the author but also related work by other people. Furthermore, the manuscripts should provide sufficient motivation, examples and applications. This clearly distinguishes Lecture Notes manuscripts from journal articles which normally are very concise. Articles intended for a journal but too long to be accepted by most journals, usually do not have this "lecture notes" character. For similar reasons it is unusual for Ph. D. theses to be accepted for the Lecture Notes series.

§ 2. Manuscripts or plans for Lecture Notes volumes should be submitted (preferably in duplicate) either to one of the series editors or to Springer- Verlag, Heidelberg . These proposals are then refereed. A final decision concerning publication can only be made on the basis of the complete manuscript, but a preliminary decision can often be based on partial information: a fairly detailed outline describing the planned contents of each chapter, and an indication of the estimated length, a bibliography, and one or two sample chapters - or a first draft of the manuscript. The editors will try to make the preliminary decision as definite as they can on the basis of the available information.

§ 3. Final manuscripts should preferably be in English. They should contain at least 100 pages of scientific text and should include

- a table of contents;
- an informative introduction, perhaps with some historical remarks: it should be accessible to a reader not particularly familiar with the topic treated;
- a subject index: as a rule this is genuinely helpful for the reader.

Further remarks and relevant addresses at the back of this book.

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B. Eckmann, Zürich
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Adviser: P. A. Meyer

Paul-André Meyer

Quantum Probability for Probabilists

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Author

Paul-André Meyer
Institut de Recherche Mathématique Avancée
Université Louis Pasteur
7, rue René Descartes
67084 Strasbourg-Cedex, France

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Introduction

These notes contain all the material accumulated over six years in Strasbourg to teach "Quantum Probability" to myself and to an audience of commutative probabilists. The text, a first version of which appeared in successive volumes of the *Séminaire de Probabilités*, has been augmented and carefully rewritten, and translated into international English. Still, it remains true "Lecture Notes" material, and I have resisted suggestions to publish it as a monograph. Being a non-specialist, it is important for me to keep the moderate right to error one has in lectures. The origin of the text also explains the addition "for probabilists" in the title : though much of the material is accessible to the general public, I did not care to redefine Brownian motion or the Ito integral.

More precisely than "Quantum Probability" , the main topic is "Quantum Stochastic Calculus" , a field which has recently got official recognition as 81S25 in the Math. Reviews classification. I find it attractive for two reasons. First, it uses the same language as quantum physics, and *does* have some relations with it, though possibly not with the kind of fundamental physics mathematicians are fond of. Secondly, it is a domain where one's experience with classical stochastic calculus is really profitable. I use as much of the classical theory as I can in the motivations and the proofs. I have tried to prepare the reader to make his way into a literature which is often hermetic, as it uses physics language and a variety of notations. I have therefore devoted much care to comparing the different notation systems (adding possibly to the general confusion with my personal habits).

It is often enlightening to interpret standard probability in a non-commutative language, and the interaction has already produced some interesting commutative benefits, among which a better understanding of classical stochastic flows, of some parts of Wiener space analysis, and above all of Wiener chaos expansions, a difficult and puzzling topic which has been renewed through Emery's discovery of the chaotic representation property of the Azéma martingales, and Biane's similar proof for finite Markov chains.

Anyone wishing to work in this field should consult the excellent book on Quantum Probability by K.R. Parthasarathy, *An Introduction to Quantum Stochastic Calculus*, as well as the seven volumes of seminars on QP edited by L. Accardi and W. von Waldenfels. It has been impossible to avoid a large overlap with Parthasarathy's monograph, all the more so, since I have myself learnt the subject from Hudson and Parthasarathy. However, I have stressed different topics, for example multiplication formulas and Maassen's kernel approach.

Our main concern being stochastic calculus on Fock space, we could not include the independent fermion approach of Barnett, Streater and Wilde, or the abstract theory of stochastic integration with respect to general "quantum martingales" (Barnett and Wilde; Accardi, Fagnola and Quaegebeur). This is unfair for historical reasons and unfortunate, since much of this parallel material is very attractive, and in need of systematic exposition. Other notable omissions are stopping times (Barnett and Wilde, Parthasarathy and Sinha), and the recent developments on "free noise" (Speicher). But also entire fields are absent from these notes : the functional analytic aspects of the dilation problem, non-commutative ergodic theory, and the discussion of concrete Langevin equations from quantum physics.

These notes also appear at a crucial time : in less than one year, there has been an impressive progress in the understanding of the analytic background of QP, and the non-commutative methods for the construction of stochastic processes are no longer pale copies of their probabilistic analogues. This progress has been taken into account, but only in part, as it would have been unreasonable to include a large quantity of still undigested (and unpublished) results.

A good part of my pleasure with QP I owe to the openmindedness of my colleagues, which behaved with patience and kindness towards a beginner. Let me mention with special gratitude, among many others, the names of L. Accardi, R.L. Hudson, J. Lewis, R. Streater, K.R. Parthasarathy, W. von Waldenfels. I owe also special thanks to S. Attal, P.D.F. Ion, Y.Z. Hu, R.L. Hudson, S. Paycha for their comments on the manuscript, which led to the correction of a large number of errors and obscurities.

P.A. Meyer, October 1992

Table of Contents