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Jean-Marc Delort

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Second Microlocalization and Semilinear Caustics

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Author

Jean-Marc Delort
Département de Mathématiques
Institut Galilée
Université Paris-Nord
Avenue J.-B. Clément
F-93430 Villetaneuse, France

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Foreword

This text grew up from lectures given at the University of Rennes I during the academic year 1988–1989. The main topics covered are second microlocalization along a lagrangian manifold, defined by Sjöstrand in [Sj], and its application to the study of conormal singularities for solutions of semilinear hyperbolic partial differential equations, developed by Lebeau [L4].

To give a quite self-contained treatment of these questions, we included some developments about FBI transformations and subanalytic geometry. The text is made of four chapters. In the first one, we define the Fourier-Bros-Iagolnitzer transformation and study its main properties. The second chapter deals with second microlocalization along a lagrangian submanifold, and with upper bounds for the wave front set of traces one may obtain using it. The third chapter is devoted to formulas giving geometric upper bounds for the analytic wave front set and for the second microsupport of boundary values of ramified functions. Lastly, the fourth chapter applies the preceding methods to the derivation of theorems about the location of microlocal singularities of solutions of semilinear wave equations with conormal data, in general geometrical situation. Every chapter begins with a short abstract of its contents, where are collected the bibliographical references.

Let me now thank all those who made this writing possible. First of all, Gilles Lebeau, from whom I learnt microlocal analysis, especially through lectures he gave with Yves Laurent at Ecole Normale Supérieure in 1982–1983. Some of the notes of these lectures have been used for the writing of parts of Chapter I. Moreover, he communicated to me the manuscripts of some of his works quoted in the bibliography before they reached their final form. Likewise, I had the possibility to consult a preliminary version of the paper of Patrick Gérard [G], where is given the characterization of Sobolev spaces in terms of FBI transformations I reproduced in Chapter one.

Moreover, this text owes much to those who attended the lectures, J. Camus, J. Chikhi, O. Guès, M. Tougeron and, especially, G. Métivier whose pertinent criticism was at the origin of many improvements of the manuscript. Lastly, let me mention that Mrs Boschet typed the french version of the manuscript, with her well known efficiency.

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Main notations

TM = tangent bundle to the manifold M.
T_xM = fiber of TM at the point x of M.
T^*M = cotangent bundle to the manifold M.
T_x^*M = fiber of T^*M at the point x of M.
T_NM = normal bundle to the submanifold N of M.
T_N^*M = conormal bundle to the submanifold N of M.
For E a vector bundle over M, $E \setminus \{0\}$ or $E \setminus 0$ denotes E minus its zero section.
For E, F two fiber bundles over M, $E \times_M F$ denotes the fibered product of E by F over M.
Over a coordinate patch of M, $E \times_M F = \{\, (x, e, f);\ e \in E_x,\ f \in F_x \,\}$.
If $h : M_1 \to M_2$ is a diffeomorphism between two manifolds, one denotes by $\tilde{h}$ the map it induces $\tilde{h} : T^*M_1 \to T^*M_2$. In local coordinates $\tilde{h}(x, \xi) = (h(x), {}^t dh(x)^{-1} \cdot \xi)$.
If $x_0 \in M_1$ and $y_0 \in M_2$, one denotes by $h : (M_1, x_0) \to (M_2, y_0)$ a germ of map from the germ of M_1 at x_0 to the germ of M_2 of y_0.
$\mathrm{gr}(\psi)$ = graph of a map ψ from a manifold to a manifold.
$d(\ ,\ )$ = euclidean (resp. hermitian) distance on the real euclidean (resp. the complex hermitian) space.
$d(\ , L)$ = distance to a subset L.
d = exterior differential on a real manifold.
∂ = holomorphic differential on a complex analytic manifold.
$\bar{\partial}$ = antiholomorphic differential on a complex analytic manifold.
$dL(x)$ = Lebesgue measure on $\mathbb{C}^n$.
We will use the standard notation for the different spaces of distributions: C_0^∞ (compactly supported smooth functions), S (Schwartz space), S' (tempered distributions), H^s (Sobolev spaces), ...

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