

Editorial Policy

§ 1. Lecture Notes aim to report new developments - quickly, informally, and at a high level. The texts should be reasonably self-contained and rounded off. Thus they may, and often will, present not only results of the author but also related work by other people. Furthermore, the manuscripts should provide sufficient motivation, examples and applications. This clearly distinguishes Lecture Notes manuscripts from journal articles which normally are very concise. Articles intended for a journal but too long to be accepted by most journals, usually do not have this “lecture notes” character. For similar reasons it is unusual for Ph. D. theses to be accepted for the Lecture Notes series.

§ 2. Manuscripts or plans for Lecture Notes volumes should be submitted (preferably in duplicate) either to one of the series editors or to Springer- Verlag, Heidelberg . These proposals are then refereed. A final decision concerning publication can only be made on the basis of the complete manuscript, but a preliminary decision can often be based on partial information: a fairly detailed outline describing the planned contents of each chapter, and an indication of the estimated length, a bibliography, and one or two sample chapters - or a first draft of the manuscript. The editors will try to make the preliminary decision as definite as they can on the basis of the available information.

§ 3. Final manuscripts should preferably be in English. They should contain at least 100 pages of scientific text and should include

- a table of contents;
- an informative introduction, perhaps with some historical remarks: it should be accessible to a reader not particularly familiar with the topic treated;
- a subject index: as a rule this is genuinely helpful for the reader.

Further remarks and relevant addresses at the back of this book.

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Similarity Problems and Completely Bounded Maps



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Foreword

These notes revolve around three similarity problems, appearing in three different contexts, but all dealing with the space $B(H)$ of all bounded operators on a complex Hilbert space H . The first one deals with group representations, the second one with C^* -algebras and the third one with the disc algebra. We describe them in detail in the introduction which follows.

This volume is devoted to the background necessary to understand these three *open* problems, to the solutions that are known in some special cases and to numerous related concepts, results, counterexamples or extensions which their investigation has generated.

For instance, we are naturally lead to study various Banach spaces formed by the matrix coefficients of group representations. Furthermore, we discuss the closely connected Schur multipliers and Grothendieck's striking characterization of those which act boundedly on $B(H)$.

While the three problems seem different, it is possible to place them in a common framework using the key concept of "*complete boundedness*", which we present in detail. In some sense, completely bounded maps can also be viewed as spaces of "coefficients" of C^* -algebraic representations, if we allow " $B(H)$ -valued coefficients", this is the content of the fundamental factorization property of these maps, which plays a central rôle in this volume.

Using this notion, the three problems can all be formulated as asking whether "boundedness" implies "complete boundedness" for linear maps satisfying certain additional algebraic identities.

Finally, a word on the structure of this book: this is definitely a lecture notes volume. Each chapter corresponds roughly to a lecture. In each one, we try to reach quickly some main point, without too many side remarks. This usually corresponds to the actual lecture. Once this point has been made, we then allow ourselves to develop all sorts of additional comments and a guide to the literature (the "notes and remarks") which expand on the first part, and which (for lack of time usually) the audience of the lecture is invited to read.

The main body of the notes is essentially self contained and can be read by anyone familiar with basic Functional and Harmonic Analysis, as presented for example in Rudin's books [R2, R4]. We believe this volume may be used as the basis for an advanced graduate course in Functional Analysis.

These notes are originally based on a course given during a summer school organized by S. Negrepointis in Spetses (Greece) in July 92, and on courses given in Texas A&M and in Paris VI in 92/93. I am very grateful to all those who subsequently provided me with more information, additional references and corrected misprints or errors of all kind. In particular, it is a pleasure to thank M. Božejko, A. Hess, J. Holbrook, C. Le Merdy, V. Mascioni, V. Paulsen, V. Peller, G. Popescu, F. Wattbled and Q. Xu.

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In conclusion, I would like to thank Robin Campbell, who typed most of the book, for her outstanding job.

Table of Contents

| | |
|---|-----|
| Foreword. | v |
| 0. Introduction. Description of contents. | 1 |
| 1. Von Neumann's inequality and Ando's generalization | 13 |
| Notes and Remarks | 28 |
| 2. Non-unitarizable uniformly bounded group representations | 30 |
| Notes and Remarks | 49 |
| 3. Completely bounded maps | 53 |
| Notes and Remarks | 69 |
| 4. Completely bounded homomorphisms and derivations | 70 |
| Notes and Remarks | 89 |
| 5. Schur multipliers and Grothendieck's inequality | 92 |
| Notes and Remarks | 105 |
| 6. Hankelian Schur multipliers. Herz-Schur multipliers | 107 |
| Notes and Remarks | 112 |
| 7. The similarity problem for cyclic homomorphisms on a C^*-algebra | 116 |
| Notes and Remarks | 131 |
| 8. Completely bounded maps in the Banach space setting | 133 |
| Notes and Remarks | 142 |
| References | 143 |
| Subject Index | 152 |
| Notation Index | 155 |