

Editorial Policy

§ 1. Lecture Notes aim to report new developments - quickly, informally, and at a high level. The texts should be reasonably self-contained and rounded off. Thus they may, and often will, present not only results of the author but also related work by other people. Furthermore, the manuscripts should provide sufficient motivation, examples and applications. This clearly distinguishes Lecture Notes manuscripts from journal articles which normally are very concise. Articles intended for a journal but too long to be accepted by most journals, usually do not have this “lecture notes” character. For similar reasons it is unusual for Ph. D. theses to be accepted for the Lecture Notes series.

§ 2. Manuscripts or plans for Lecture Notes volumes should be submitted (preferably in duplicate) either to one of the series editors or to Springer- Verlag, Heidelberg . These proposals are then refereed. A final decision concerning publication can only be made on the basis of the complete manuscript, but a preliminary decision can often be based on partial information: a fairly detailed outline describing the planned contents of each chapter, and an indication of the estimated length, a bibliography, and one or two sample chapters - or a first draft of the manuscript. The editors will try to make the preliminary decision as definite as they can on the basis of the available information.

§ 3. Final manuscripts should preferably be in English. They should contain at least 100 pages of scientific text and should include

- a table of contents;
- an informative introduction, perhaps with some historical remarks: it should be accessible to a reader not particularly familiar with the topic treated;
- a subject index: as a rule this is genuinely helpful for the reader.

Further remarks and relevant addresses at the back of this book.

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Springer-Verlag
Berlin Heidelberg GmbH

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Integrable Systems in the realm of Algebraic Geometry



Springer

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Cataloging-in-Publication Data applied for

Die Deutsche Bibliothek - CIP-Einheitsaufnahme

Vanhaecke, Pol:

Integrable systems in the realm of algebraic geometry / Pol
Vanhaecke.

(Lecture notes in mathematics ; 1638)

ISBN 978-3-540-61886-7

ISBN 978-3-662-21535-7 (eBook)

DOI 10.1007/978-3-662-21535-7

NE: GT

Mathematics Subject Classification (1991): 14K20, 35Q58, 58F07

ISSN 0075-8434

ISBN 978-3-540-61886-7

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Originally published by Springer-Verlag Berlin Heidelberg New York 1996

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Typesetting: Camera-ready T_E X output by the author

SPIN: 10479887

46/3142-543210 - Printed on acid-free paper

Acknowledgments

The help of many people was indispensable for establishing and presenting the results which are contained in this work. Not enough credit can be given to those who created at home, at the Max-Planck-Institut in Bonn, at the University of Lille and finally at the University of California at Davis a pleasant and stimulating atmosphere. Even some people I don't know by name should be thanked here.

Special thanks are due to Mark Adler and Pierre van Moerbeke, whose fundamental work on a.c.i. systems was the starting point for the research contained in this book. Stimulating discussions with them have led to an improvement of many of the results and to a better understanding of the subject. Also Michèle Audin deserves a special place here for sharing her insights with me through long discussions and letters. Extremely helpful for a thorough understanding were several algebraic-geometric explanations by Laurent Gruson.

I wish to thank my collaborators José Bertin and Marco Pedroni for a fruitful interaction. I have also benefited from discussions with my colleagues at Lille, in particular Jean d'Almeida, Robert Gergondey, Johannes Huebschmann, Raphaël Freitas, Armando Treibich, Gijs Tuynman and Alberto Verjowski and at UC Davis, in particular Josef Mattes, Motohiko Mulase, Michael Penkava, Albert Schwarz and Craig Tracy.

I also acknowledge my other friends scattered around the globe, to wit, Christina Birkenhake, Robert Brouzet, Peter Bueken, Jan Denef, Paul Dhooghe, Jean Fastré, Ljubomir Gavrilov, Luc Haine, Horst Knörrer, Franco Magri, Askold Perelomov, Luis Piovan, Elisa Prato and Taka Shiota for their interest in my work and helpful related discussions.

For useful comments on the manuscript I am indebted to Michèle Audin, an anonymous referee and several students in my graduate course in UC Davis (Spring 1996).

Last but not least, special thanks to my wife Lieve for her constant assistance through this adventure.

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