

# Texts and Monographs in Physics

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*Series Editors:* R. Balian W. Beiglböck H. Grosse E. H. Lieb  
N. Reshetikhin H. Spohn W. Thirring

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# Scattering Theory of Classical and Quantum $N$ -Particle Systems

With 15 Figures



Springer

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Ida, Line, Marion, Michał and Pierre*

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