

Die Grundlehren der mathematischen Wissenschaften

in Einzeldarstellungen
mit besonderer Berücksichtigung
der Anwendungsgebiete

Band 198

Herausgegeben von J. L. Doob A. Grothendieck E. Heinz
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*Geschäftsführende
Herausgeber* B. Eckmann und B. L. van der Waerden

Steven A. Gaal

Linear Analysis
and Representation
Theory



Springer-Verlag
Berlin Heidelberg New York 1973

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Herausgeber

B. Eckmann
Eidgenössische Technische Hochschule Zürich

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Mathematisches Institut der Universität Zürich

AMS Subject Classifications (1970)

Primary: 22 D 05, 22 D 10, 22 D 15, 22 D 25, 22 D 30, 22 E 15,
22 E 60, 28 A 70, 43 A 35, 43 A 65, 43 A 85, 43 A 90

Secondary: 46 H 05, 46 J 05, 47 A 10, 47 B 05, 47 B 10, 58 A 05

ISBN-13:978-3-642-80743-5

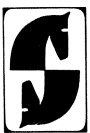
e-ISBN-13:978-3-642-80741-1

DOI: 10.1007/978-3-642-80741-1

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Softcover reprint of the hardcover 1st edition 1973

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Softcover reprint of the hardcover 1st edition 1973

Preface

In an age when more and more items are made to be quickly disposable or soon become obsolete due to either progress or other man caused reasons it seems almost anachronistic to write a book in the classical sense. A mathematics book becomes an indispensable companion, if it is worthy of such a relation, not by being rapidly read from cover to cover but by frequent browsing, consultation and other occasional use. While trying to create such a work I tried not to be encyclopedic but rather select only those parts of each chosen topic which I could present clearly and accurately in a formulation which is likely to last. The material I chose is all mathematics which is interesting and important both for the mathematician and to a large extent also for the mathematical physicist. I regret that at present I could not give a similar account on direct integrals and the representation theory of certain classes of Lie groups. I carefully kept the level of presentation throughout the whole book as uniform as possible. Certain introductory sections are kept shorter and are perhaps slightly more detailed in order to help the newcomer progress with it at the same rate as the more experienced person is going to proceed with his study of the details. On the one hand I made the chapters as independently readable as possible under the given conditions and on the other hand I gave as many cross references as seemed practical in order to increase the serviceability of the volume and reinforce its unity.

I offer my grateful thanks for the encouragement given to me by my mother, wife and daughters, Barbara and Dorothy during the years when this book was being written. I also thank the farmers, workers and industries of Minnesota whose taxes supported myself and my family during these years through the tenure of a professorship at their University.

My appreciation is due to Professor Robert Fakler of the University of Michigan who read almost the entire manuscript very carefully and pointed out a number of minor inaccuracies and many errors and omissions made by me while typing the manuscript and filling in the various symbols by hand. His voluntary efforts substantially improved the quality of the manuscript.

My sincere thanks to Springer-Verlag and the various people associated with it for the encouragement and help during the actual writing and for the efficient and accurate work during the production. I like to thank especially Professor B. Eckmann and Dr. K. Peters for their constant interest and gentle persuasion during the past years. I also thank Dr. E. Pannwitz of Zentralblatt für Mathematik who first recommended my works and brought me into contact with the publisher of the Grundlehren der mathematischen Wissenschaften series.

The employees of the Zehnersche Buchdruckerei in Speyer did an outstanding work during the typesetting and printing of this volume. It is a pleasure to express my appreciation for their cooperation and accuracy.

Finally I thank Dr. K.F. Springer for further strengthening my past association with Springer-Verlag and for promoting mathematics by publishing what follows.

Minneapolis, January 27, 1973

Steven Alexander Gaal

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