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Discontinuous Galerkin Methods

Theory, Computation
and Applications

With 138 Figures



Springer

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Preface

A class of finite element methods, the Discontinuous Galerkin Methods (DGM), has been under rapid development recently and has found its use very quickly in such diverse applications as aeroacoustics, semi-conductor device simulation, turbomachinery, turbulent flows, materials processing, MHD and plasma simulations, and image processing. While there has been a lot of interest from mathematicians, physicists and engineers in DGM, only scattered information is available and there has been no prior effort in organizing and publishing the existing volume of knowledge on this subject.

In May 24-26, 1999 we organized in Newport (Rhode Island, USA), the first international symposium on DGM with equal emphasis on the theory, numerical implementation, and applications. Eighteen invited speakers, leaders in the field, and thirty-two contributors presented various aspects and addressed open issues on DGM. In this volume we include forty-nine papers presented in the Symposium as well as a survey paper written by the organizers. All papers were peer-reviewed. A summary of these papers is included in the survey paper, which also provides a historical perspective of the evolution of DGM and its relation to other numerical methods.

We hope this volume will become a major reference in this topic. It is intended for students and researchers who work in theory and application of numerical solution of convection dominated partial differential equations. The papers were written with the assumption that the reader has some knowledge of classical finite elements and finite volume methods.

Finally, we would like to acknowledge the financial support by the National Science Foundation, the Department of Energy, and the Army Research Office. We especially like to thank Ms. Madeline Brewster who has organized the Symposium, collected the papers, typeset this volume, and made this first symposium on DGM a success.

July 1999

The Organizers
Bernardo Cockburn
George Em Karniadakis
Chi-Wang Shu

Table of Contents

Part I Overview

The Development of Discontinuous Galerkin Methods <i>Bernardo Cockburn, George E. Karniadakis, and Chi-Wang Shu</i>	3
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Part II Invited Papers

Steps Toward a Robust High-Order Simulation Tool for Aerospace Applications <i>Harold L. Atkins</i>	53
Simplified Discontinuous Galerkin Methods for Systems of Conservation Laws with Convex Extension <i>Timothy J. Barth</i>	63
A High Order Discontinuous Galerkin Method for Compressible Turbulent Flows <i>Francesco Bassi and Stefano Rebay</i>	77
Discontinuous Galerkin Methods for Elliptic Problems <i>Douglas N. Arnold, Franco Brezzi, Bernardo Cockburn, and Donatella Marini</i>	89
Analysis of Finite Element Methods for Linear Hyperbolic Problems <i>Richard S. Falk</i>	103
Software for the Parallel Adaptive Solution of Conservation Laws by Discontinuous Galerkin Methods <i>Joseph E. Flaherty, Raymond M. Loy, Mark S. Shephard, and Jim D. Teresco</i>	113
Simulation of Gravity Flow of Granular Materials in Silos <i>Pierre A. Gremaud and John V. Matthews</i>	125

VIII Table of Contents

A Comparison of Discontinuous and Continuous Galerkin Methods Based on Error Estimates, Conservation, Robustness and Efficiency
Thomas J.R. Hughes, Gerald Engel, Luca Mazzei, and Mats G. Larson 135

The Utility of Modeling and Simulation in Determining Transport Performance Properties of Semiconductors
Bernardo Cockburn, Joseph W. Jerome, and Chi-Wang Shu 147

A Discontinuous Galerkin Method for the Incompressible Navier-Stokes Equations
Ohannes Karakashian and Theodoros Katsaounis 157

Full Convergence for Hyperbolic Finite Elements
Qun Lin 167

A Conservative DGM for Convection-Diffusion and Navier-Stokes Problems
J. Tinsley Oden and Carlos Erik Baumann 179

GMRES Discontinuous Galerkin Solution of the Compressible Navier-Stokes Equations
Francesco Bassi and Stefano Rebay 197

Explicit Finite Element Methods for Linear Hyperbolic Systems
Richard S. Falk and Gerard R. Richter 209

hp-DGFEM for Partial Differential Equations with Nonnegative Characteristic Form
Endre Süli, Christoph Schwab, and Paul Houston 221

A Discontinuous Galerkin Method Applied to Nonlinear Parabolic Equations
Béatrice Rivière and Mary F. Wheeler 231

Part III Contributed Papers

Parallel Iterative Discontinuous Galerkin Finite-Element Methods
Dan Aharoni and Amnon Barak 247

A Discontinuous Projection Algorithm for Hamilton Jacobi Equations
Steeve Augoula and Rémi Abgrall 255

Successes and Failures of Discontinuous Galerkin Methods in Viscoelastic Fluid Analysis <i>Arjen C.B. Bogaerds, Wilco M.H. Verbeeten, and Frank P.T. Baaijens</i>	263
High Order Current Basis Functions for Electromagnetic Scattering of Curved Surfaces <i>Wei Cai</i>	271
An Adaptive Discontinuous Galerkin Model for Coupled Viscoplastic Crack Growth and Chemical Transport <i>Fernando L. Carranza and R.B. Haber</i>	277
An Optimal Estimate for the Local Discontinuous Galerkin Method <i>Paul Castillo</i>	285
Post-Processing of Galerkin Methods for Hyperbolic Problems <i>Bernardo Cockburn, Mitchell Luskin, Chi-Wang Shu, and Endre Süli</i> . .	291
Introduction to Discontinuous Wavelets <i>Nicholas Coult</i>	301
The Local Discontinuous Galerkin Method for Contaminant Transport Problems <i>Clint Dawson, Vadym Aizinger, and Bernardo Cockburn</i>	309
Discontinuous Galerkin Method for the Numerical Solution of Euler Equations in Axisymmetric Geometry <i>Bruno Despres</i>	315
Ten Years Using Discontinuous Galerkin Methods for Polymer Processing Problems <i>André Fortin, Alain Béliveau, Marie-Claude Heuzey, and Alain Lioret</i>	321
Using Krylov-Subspace Iterations in Discontinuous Galerkin Methods for Nonlinear Reaction-Diffusion Systems <i>Donald J. Estep and Roland W. Freund</i>	327
An Abridged History of Cell Discretization <i>John Greenstadt</i>	337
The Effect of the Least Square Procedure for Discontinuous Galerkin Methods for Hamilton-Jacobi Equations <i>Changqing Hu, Olga Lepsky, and Chi-Wang Shu</i>	343

A Posteriori Error Estimate in the Case of Insufficient Regularity
of the Discrete Space
Guido Kanschat and Franz-Theo Suttmeier 349

Discontinuous Spectral Element Approximation of Maxwell's Equations
David A. Kopriva, Stephen L. Woodruff, and M.Y. Hussaini 355

A Posteriori Error Estimation for Adaptive Discontinuous Galerkin
Approximations of Hyperbolic Systems
Mats G. Larson and Timothy J. Barth 363

A Numerical Example on the Performance of High Order Discontinuous
Galerkin Method for 2D Incompressible Flows
Jian-Guo Liu and Chi-Wang Shu 369

A Discontinuous Galerkin Method in Moving Domains
Igor Lomteu, Robert M. Kirby, and George E. Karniadakis 375

Discontinuous Galerkin for Hyperbolic Systems with Stiff Relaxation
Robert B. Lowrie and Jim E. Morel 385

Finite Element Output Bounds for Parabolic Equations:
Application to Heat Conduction Problems
Luc Machiels 391

3D Unstructured Mesh ALE Hydrodynamics
with the Upwind Discontinuous Galerkin Method
*Manoj K. Prasad, Jose L. Milovich, Aleksei I. Shestakov,
David S. Kershaw and Michael J. Shaw* 397

Some Remarks on the Accuracy of a Discontinuous Galerkin Method
Patrick Rasetarinera, Mohammed Y. Hussaini, and Fang Q. Hu 407

Coupling Continuous and Discontinuous Techniques:
An Adaptive Approach
Mirko Sardella 413

A Discontinuous Galerkin Method for the Shallow Water Equations
with Source Terms
Dirk Schwanenberg and Jürgen Köngeter 419

Dispersion Analysis of the Continuous
and Discontinuous Galerkin Formulations
Spencer Sherwin 425

The Cell Discretization Algorithm; An Overview <i>Howard Swann</i>	433
Accuracy, Resolution, and Computational Complexity of a Discontinuous Galerkin Finite Element Method <i>Harmen van der Ven and J.J.W. van der Vegt</i>	439
An ELLAM Scheme for Porous Medium Flows <i>Hong Wang</i>	445
Application of the Discontinuous Galerkin Method to Maxwell's Equations Using Unstructured Polymorphic hp-Finite Elements <i>Tim Warburton</i>	451
A Space-Time Discontinuous Galerkin Method for Elastodynamic Analysis <i>Lin Yin, Amit Acharya, Nahil Sobh, Robert B. Haber, and Daniel A. Tortorelli</i>	459
Nonconforming, Enhanced Strain, and Mixed Finite Element Methods – A Unified Approach <i>Zhimin Zhang</i>	465