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Editors

Recent Developments in Domain Decomposition Methods

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Cover figure: Convective flow with an internal dissipative circular region. Discretization with linear finite elements on non-matching grids and a discontinuous Galerkin approach by A. Toselli

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Preface

This volume collects some of the papers presented at the Workshop on Domain Decomposition held at ETH, Zürich, on June 7-8th 2001. The Workshop was organized by Luca F. Pavarino (University of Milan), Christoph Schwab (ETH Zürich), Andrea Toselli (ETH Zürich), and Olof B. Widlund (Courant Institute of Mathematical Sciences). Our sponsors were the University of Milan, Department of Mathematics (MURST projects: "Calcolo Scientifico: modelli e metodi numerici innovativi" and "Simmetrie, forme geometriche, evoluzione e memoria nelle equazioni alle derivate parziali"), the Seminar for Applied Mathematics, ETH Zürich, and the Program on Computational Science and Engineering at ETH Zürich.

The main goal of this meeting was to provide a forum for the exchange of ideas on the most recent developments in the field of Domain Decomposition Methods. We broadly understand Domain Decomposition as relating to the construction of preconditioners for the large algebraic systems of equations which often arise in applications, by solving smaller instances of the same problem. In our planning, we also wished to include studies of methods built from different discretizations in different subdomains such as in multi-physics models, mortar finite elements, wavelets, etc. Domain Decomposition methods are now fairly well understood for elliptic scalar and vector problems and are employed for the solution of large scale problems in computational sciences and engineering. However they remain less well understood for more general problems, such as scattering problems, mixed problems, wave propagation, and evolution problems. In addition, even for elliptic equations, some delicate important issues still need to be fully addressed, such as the improvement of some of the particular components of Domain Decomposition methods (coarse and local solvers) and their efficient application to a larger class of approximation methods (*hp*, spectral and wavelet approximations).

Among the most successful Domain Decomposition algorithms, we mention the Overlapping Schwarz, Neumann–Neumann and FETI (Finite Element Tearing and Interconnecting) methods. While the first class of methods is based on the solution of local problems on overlapping subdomains, Neumann–Neumann and FETI methods rely on a non-overlapping partition into subdomains (substructures). In a Neumann–Neumann method, a preconditioner is built with a low-dimensional coarse global problem, with a few degrees of freedom associated to each subdomain, and local Neumann problems on the subdomains. In a FETI method, the discrete problem is reformulated imposing the continuity of the finite element solution across the interface between the subdomains by introducing Lagrange multipliers. The primal variables are then implicitely eliminated, yielding an equation for the Lagrange multipliers. The continuity across the interface of the underlying finite element method is only fully satisfied at the convergence of the iteration. A FETI preconditioner is also built using local problems on the substructures. More recently, dual–primal FETI methods have been introduced. Here, Lagrange multipliers are still used but a few select interface continuity constraints are enforced in each iteration for particular sets of degrees of freedom on the interface. These new algorithms offer a number of advantages especially for very large and heterogeneous problems.

In mortar approximations, independent discretization methods can be employed in different subdomains. While the basic theory is quite well understood for elliptic problems, the application of mortar methods to more general problems is an ongoing field of research. Once the approximation properties of a mortar method are understood, one is left with the task of solving the corresponding linear system. If Lagrange multipliers are associated to the weak-continuity constraints across the interface between the subdomains, a mixed problem, formally the same as that employed in FETI methods, is obtained. It is then natural to generalize FETI preconditioners to linear systems arising from mortar approximations. Such generalizations are however far from straightforward.

The solution of a coarse problem is usually necessary in order to obtain scalable Domain Decomposition preconditioners, i.e., methods with a convergence rate that does not deteriorate with an increasing number of subdomains. The use of a coarse mesh is often quite non-trivial in particular when unstructured meshes are employed. An alternative is to construct coarse spaces that are not directly associated with a coarse mesh. In partition of unity and smoothed aggregation coarse spaces, the degrees of freedom can be associated instead with single subdomains of an overlapping partition and coarse basis functions can be properly constructed.

As a general introduction to Domain Decomposition methods we refer to the books by B. F. Smith, P.Bjørstad, and W. D. Gropp, *Domain Decomposition: Parallel Multilevel Methods for Elliptic Partial Differential Equations*, Cambridge University Press, 1996 and by A. Quarteroni and A. Valli, *Domain Decomposition Methods for Partial Differential Equations*, Oxford Science Publications, 1999. At the official Domain Decomposition web site http://www.ddm.org, interested readers can find information about Domain Decomposition meetings, proceedings and other related material.

About thirty scientists, from Europe and the United States, participated in the Workshop. This collection consists of fourteen of the twenty-one papers presented at the Workshop and their topics reflect some of the most active research areas in Domain Decomposition, such as:

- the development and analysis of novel FETI methods and Neumann-Neumann methods for the solution of systems arising from the approximations of partial differential equations (see Hetmaniuk and Farhat, Klawonn et al., Dryja and Widlund, and Goldfeld et al.);

- the construction and analysis of coarse solvers for two-level overlapping methods that do not require the introduction of a coarse triangulation (see Sarkis, and Lasser and Toselli);
- mortar methods for approximations on non-matching grids (see Maday et al., Bertoluzza et al., and Ben Belgacem et al.);
- preconditioners for scalar and vector scattering problems (see Lai et al., Hetmaniuk and Farhat, and Alonso and Valli);
- *hp* approximations and their efficient solution by iterative substructuring methods (see Bauer et al., Ben Belgacem et al.);
- block ILU preconditioners based on iterated filtering decompositions (see Achdou and Nataf);
- Domain Decomposition in time for evolution problems (see Bal and Maday).

We wish to thank Christoph Schwab and Olof Widlund for their help in organizing the Workshop.

Milan, Zürich,
March 2002

Luca F. Pavarino
Andrea Toselli

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