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Dipartimento di Energetica "S. Stecco"

Università di Firenze

Via S. Marta, 3

50139 Florence

Italy

e-mail: zecca@unifi.it

C.I.M.E. Secretary

Elvira MASCOLO

Dipartimento di Matematica "U. Dini"

Università di Firenze

viale G.B. Morgagni 67/A

50134 Florence

Italy

e-mail: mascolo@math.unifi.it

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Lou van den Dries • Jochen Koenigsmann •
H. Dugald Macpherson • Anand Pillay •
Carlo Toffalori • Alex J. Wilkie

Model Theory in Algebra, Analysis and Arithmetic

Cetraro, Italy 2012

Editors: H. Dugald Macpherson,
Carlo Toffalori



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CENTRO INTERNAZIONALE MATEMATICO ESTIVO
INTERNATIONAL MATHEMATICAL SUMMER CENTER

Lou van den Dries
Department of Mathematics
University of Illinois
Urbana, IL, USA

Jochen Koenigsmann
Mathematical Institute
University of Oxford
Oxford, United Kingdom

H. Dugald Macpherson
University of Leeds School of Mathematics
Leeds, United Kingdom

Anand Pillay
Department of Mathematics
University of Notre Dame
Hurley, IN, USA

Carlo Toffalori
Division of Mathematics
University of Camerino School of Science
and Technology
Camerino, Italy

Alex J. Wilkie
University of Manchester School
of Mathematics
Manchester, United Kingdom

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Contents

Model Theory in Algebra, Analysis and Arithmetic: A Preface	1
Dugald Macpherson and Carlo Toffalori	
Some Themes Around First Order Theories Without the Independence Property	13
Anand Pillay	
Lectures on the Model Theory of Real and Complex Exponentiation	35
A.J. Wilkie	
Lectures on the Model Theory of Valued Fields	55
Lou van den Dries	
Undecidability in Number Theory	159
Jochen Koenigsmann	