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Peter Constantin • Arnaud Debussche
Giovanni P. Galdi • Michael Růžička
Gregory Seregin

Topics in Mathematical Fluid Mechanics

Cetraro, Italy 2010

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Preface

Fluid mechanics is one of the most active areas of science, at the intersection of mathematics, physics, geophysics, and engineering and, inside mathematics, the activity is spread between several fields, such as analysis, scientific computing, probability, and geometry. The series of lectures delivered at the CIME school on "Topics in mathematical fluid mechanics," in Cetraro, September 2010, was mainly devoted to foundational issues but with a wide variety of models and applications in mind. The Navier–Stokes equations for an incompressible Newtonian fluid are the paradigm of any investigation in fluid mechanics and almost all lectures reviewed recent progresses on them. But several variants and generalizations of interest for mathematics and applied sciences have been treated as well, as generalized Newtonian fluids, fluids interacting with particles and with solids, models in geophysics, and stochastic models. The questions addressed in the lectures ranged from the basic problems of existence of weak and more regular solutions, the local regularity theory and analysis of potential singularities, qualitative and quantitative results about the behavior in special cases, asymptotic behavior, statistical properties, and ergodicity.

Beirao da Veiga
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A Brief Content of the Lectures

This volume contains five lectures, by Peter Constantin of Chicago University, Arnaud Debussche of E.N.S. Cachan at Rennes, Giovanni P. Galdi of Pittsburgh University, Michael Růžička of Freiburg University, and Gregory A. Seregin of Oxford University.

Peter Constantin's lectures were devoted to the mathematical descriptions of complex fluids, in which particles and fluids interact. The particles are carried by

the fluid, are thermally agitated, and interact among themselves. The particles add stress to the fluid system and drive the fluids. The thermodynamic equilibrium of the particles is described by Onsager's equation, a nonlocal nonlinear equation for the probability distribution of particles on a configuration space. The kinetic description of interacting particles is done via certain nonlinear Fokker–Planck equations, the Smoluchowski equations. The lessons were concerned with mathematical questions of existence, stability, asymptotic behavior, and traveling waves.

Arnaud Debussche's lectures were devoted to the stochastic Navier–Stokes equations. After some preliminaries, the main body of the lectures was devoted to ergodicity, treated at different levels of generality on the degeneracy of the noise. Different approaches have been presented: one suitable in the strictly dissipative case (high viscosity) and one in the nondegenerate case; then the more refined methods, based also on Malliavin calculus, to deal with degenerate noise, first in the case of a large number of excited modes, then in the very degenerate noise. Finally the 3D theory has been discussed, where ergodicity is based on a preliminary analysis of the Kolmogorov equation.

Giovanni P. Galdi's lectures were devoted to mathematical problems of fluid–solid interaction. Topics include non-Newtonian fluid models, nonlinear elasticity, and multiphase flow. Part of the lectures gave conditions under which the governing equations possess the fundamental requirements of well-posedness. Another part of more applied nature showed that these models give a satisfactory interpretation of the observed phenomena. The motion of a rigid body in Newtonian and viscoelastic liquid, the steady flow of a Newtonian fluid past an elastic body, and the self-propelled motion of a deformable body in a Newtonian liquid are among the considered examples.

Michael Růžička lectures were devoted to the analysis of generalized Newtonian fluids, namely fluids not described by a linear constitutive law (Newtonian fluids) but by power law like constitutive relation. The existence theory for weak and strong solutions, as well as the numerical analysis of appropriate approximations, including space and time discretizations, have been the main subjects. The Lipschitz truncation method was presented. Error analysis for space and time discretizations of the equations has been the last main topic of these lectures.

Gregory A. Seregin's lectures were devoted to local regularity theory for the three-dimensional nonstationary Navier–Stokes equations. Such a theory differs essentially from the usual local regularity theory for the classical PDE's of elliptic and parabolic types. The so-called Caffarelli–Kohn–Nirenberg type theory for suitable weak solutions was discussed, based on the Navier–Stokes scaling and scaled-invariant functionals. The recent construction of ancient solutions to the Navier–Stokes equations with different kinds of blowup at possible singular points was given, including problems of unique continuation and Liouville's theorems for those ancient solutions.

A sixth course entitled "Mathematical Analysis of Certain Geophysical Models and Sub-grid Scale Models of Turbulence" held at the workshop by Edriss S. Titi of the University of California is not included in this volume. We wish to thank all

lecturers and participants for their contribution to the success of the school, CIME scientific committee for giving us the opportunity to organize this event, and CIME staff for their efficient help.

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