Lecture Notes in Mathematics 2072

For further volumes:
http://www.springer.com/series/304

Igor Reider

Nonabelian Jacobian of Projective Surfaces

Geometry and Representation Theory

Springer

Igor Reider
Université d'Angers
Angers, France

ISBN 978-3-642-35661-2 ISBN 978-3-642-35662-9 (eBook)
DOI 10.1007/978-3-642-35662-9
Springer Heidelberg New York Dordrecht London

Lecture Notes in Mathematics ISSN print edition: 0075-8434
ISSN electronic edition: 1617-9692

Library of Congress Control Number: 2013932343

Mathematics Subject Classification (2010): 14J60, 14C05, 16G30

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

The monograph studies representation theoretic aspects of a nonabelian version of the Jacobian for a smooth complex projective surface X introduced in [R1]. The sheaf of reductive Lie algebras $\tilde{\mathcal{G}}$ associated to the nonabelian Jacobian is determined and its Lie algebraic properties are explicitly related to the geometry of configurations of points on X. In particular, it is shown that the subsheaf of centres of $\tilde{\mathcal{G}}$ determines a distinguished decomposition of configurations into the disjoint union of subconfigurations. Furthermore, it is shown how to use $\mathbf{sl_2}$-subalgebras associated to certain nilpotent elements of $\tilde{\mathcal{G}}$ to write equations defining configurations of X in appropriate projective spaces.

The same nilpotent elements are used to establish a relation of the nonabelian Jacobian with such fundamental objects in the representation theory as nilpotent orbits, Springer resolution and Springer fibres of simple Lie algebras of type $\mathbf{A_n}$, for appropriate values of n. This leads to a construction of distinguished collections of objects in the category of representations of symmetric groups as well as in the category of perverse sheaves on the appropriate Hilbert schemes of points of X. Hence two ways of categorifying the second Chern class of vector bundles of rank 2 on smooth projective surfaces.

We also give a "loop" version of the above construction by relating the nonabelian Jacobian to the infinite Grassmannians of simple Lie groups of type $\mathbf{SL_n(C)}$, for appropriate values of n. This gives, via the geometric version of the Satake isomorphism, a distinguished collection of irreducible representations of the Langlands dual groups thus indicating a relation of the nonabelian Jacobian to the Langlands duality for smooth projective surfaces.

All of the above constitutes a substantial body of evidence that our nonabelian Jacobian provides an efficient mechanism for using the representation theory of reductive Lie algebras/groups in a *systematic* way to gain insight into various algebro-geometric properties of smooth complex projective surfaces.

Angers, France Igor Reider

Contents