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Stochastic Geometry, Spatial Statistics and Random Fields

Asymptotic Methods

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. . . Geometry is the knowledge of the eternally existent.

Plato, "Republic", 527.

Foreword

Geometric intuition is central to many areas of statistics and probabilistic arguments. This is particularly true in the areas covered by this book, namely stochastic geometry, random fields and random geometric graphs. Nevertheless, intuition must be followed with rigorous arguments if it is to become part of the general literature of probability and statistics. Many such arguments in this area deviate from traditional statistics, requiring special (and often beautiful) tools outside a working statistician's usual toolbox.

Professor Spodarev assembled a very impressive cast of instructors for a workshop in Söllerhaus in September 2009 in order to further the literature in this area and to introduce the topics to participating graduate students. I previously had the pleasure of hosting several of the contributors of this volume at workshop at BIRS in February 2009 on Random Fields and Stochastic Geometry and am certain the workshop in Söllerhaus was a tremendous success.

The success of the workshop is further evidenced from this volume of lecture notes. Professor Spodarev has managed to produce a volume that combines both introductory material and current research in these notes. This book will be a useful reference for myself in this area, as well as to all researchers with an interest in stochastic geometry.

Stanford, CA
March 2012

Jonathan Taylor

Preface

This volume is an attempt to write a textbook providing a modern introduction to stochastic geometry, spatial statistics, the theory of random fields and related topics. It has been made out of selected contributions to the Summer Academy on Stochastic Geometry, Spatial Statistics and Random Fields

http://www.uni-ulm.de/summeracademy09

which took place during 13–26 Sep 2009 at Söllerhaus, an Alpine conference centre of the University of Stuttgart and RWTH Aachen, in the village Hirschegg (Austria). It was organized by the Institute of Stochastics of Ulm University in cooperation with the Chair of Probability Theory of Lomonosov Moscow State University. In contrast with previous schools on this subject (Sandbjerg 2000, Martina Franca 2004, Sandbjerg 2007), this summer academy concentrated on the asymptotic theory of random sets, fields and geometric graphs. At the same time, it provided an introduction to more classical subjects of stochastic geometry and spatial statistics, giving (post)graduate students an opportunity to start their own research within a couple of weeks. The summer academy hosted 38 young participants from 13 countries (Australia, Austria, Denmark, Germany, France, Mongolia, Russia, Romania, Sweden, Switzerland, UK, USA and Vietnam). Twelve experts gave lectures on various domains of geometry, probability theory and mathematical statistics. Moreover, students and young researchers had the possibility to give their own short talks.

As it was pointed out above, this volume is focused on the asymptotic methods in the theory of random geometric objects (point patterns, sets, graphs, trees, tessellations and functions). It reflects advances in this domain made within the last two decades. This especially concerns the limit theorems for random tessellations, random polytopes, finite point processes and random fields.

The book is organized as follows. The first chapter provides an introduction to the theory of random closed sets (RACS). It starts with the foundations of geometric probability (Buffon needle problem, Bertrand's paradox) and continues with the classical theory of random sets by Matheron. Then it gives laws of large numbers and limit theorems for Minkowski sums and unions of independent

identically distributed (i.i.d.) RACS. Chapter 2 provides basics of the classical integral geometry and its applications to stereology, a part of spatial stochastics which deals with the reconstruction of the higher-dimensional properties of geometric objects from lower-dimensional sections. In Chap. 3, principal classes of spatial point processes (Poisson-driven point processes, finite point processes) are introduced. Their simulation and statistical inference techniques (partially using the Markov Chain Monte Carlo (MCMC) methods) are discussed as well. Chapter 4 provides an account of the theory of marked point processes and the asymptotic statistics for them in growing domains. Ergodicity, mixing and m-dependence properties of marked point processes are studied in detail. Random tessellation models are the matter of Chap. 5. There, Poisson-driven tessellations as well as Cox processes on them and hierarchical networks constructed on their basis are considered. Scaling limits for some characteristics of these networks are found. Applications to telecommunication networks are also discussed. Distribution tail asymptotics and limit theorems for the characteristics of the (large) typical cell of Poisson hyperplane and Poisson–Voronoi tessellations are given in Chap. 6. The shape of large cells of hyperplane tessellations as well as limit theorems for some geometric functionals of convex hulls of a large number of i.i.d. points within a convex body and of random polyhedra are dealt with in Chap. 7. Weak laws of large numbers and central limit theorems for functionals of finite point patterns are discussed in Chap. 8. Additionally, their applications to various topics ranging from optimization to sequential packings of convex bodies are touched upon. Chapter 9 surveys the elementary theory of random functions with the focus on random fields. Basic classes of random field models as well as an account of the correlation theory, statistical inference and simulation techniques are provided. Special attention is paid to infinitely divisible random functions. Dependence concepts for random fields (such as mixing, association, (BL, θ)-dependence) as well as central limit theorems for weakly dependent random fields are the subject of Chap. 10. They are applied to establish the limiting behaviour of the volume of excursion sets of weakly dependent stationary random fields. Chapter 11 focuses on almost sure limit theorems for partial sums (or increments) of random fields on $\mathbb{N}^d$ such as laws of large numbers, laws of single or iterated logarithm and others. In the final chapter, the geometry of large rooted plane random trees with nearest neighbour interaction is studied. A law of large numbers, a large deviation principle for the branching type statistics and scaling limits of the tree are considered. Connections of these results with the solutions of some partial differential equations are discussed as well. Some of the chapters are written in a more formal and rigorous way than others which reflects the personal taste and style of the authors.

The topics of this volume are (almost) self-contained. Thus, we recommend Chaps. 1, 2 and 7 for the first acquaintance with the theory of random sets. A reader interested in the (asymptotic theory of) point processes might start reading with Chap. 3 and continue with Chaps. 4, 5 and 8 following the references to Chap. 1 if needed. Readers with an interest in tessellations and random polytopes might additionally read Chaps. 2, 6 and 7. To get a concise introduction to random fields and limit theorems for them, one could read Chaps. 9–11 occasionally following the

references to earlier chapters. For random graphs and trees, Chaps. 8 and 12 are a good starting point.

All in all, the authors hope that the present volume will be helpful to graduates and PhD students in mathematics to get a first glance of the geometry of random objects and its asymptotical methods. Written in the spirit of a textbook (with a significant number of proofs and exercises for active reading), it might be also instrumental to lecturers in preparing their own lecture courses on this subject.

Söllerhaus at Hirschegg Evgeny Spodarev
September 2011

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Acronyms

Sets

$\mathbb{C}$	Complex numbers
$\mathbb{N}$	Positive integer numbers
$\mathbb{Q}$	Rational numbers
$\mathbb{R}$	Real numbers
$\mathbb{Z}$	All integer numbers
$\mathcal{K}^d$	Class of all compact sets
$\mathcal{K}^d_{\text{conv}}$	Class of all convex bodies in $\mathbb{R}^d$
$\mathcal{E}^d_k$	Family of affine k-planes in $\mathbb{R}^d$
$\mathcal{B}_0(\mathbb{R}^d)$	Family of bounded Borel sets in $\mathbb{R}^d$
$\mathbb{S}^{d-1}$	Unit sphere in $\mathbb{R}^d$

Probability Theory

ξ, η	Random variables
X, Y	Random sets
$\mathbf{P}_\xi$	Probability measure of ξ
$\mathbf{E}\,\xi$	Expectation of ξ
$\mathbf{corr}(\xi, \eta)$	Correlation of ξ and η
$\mathbf{cov}(\xi, \eta)$	covariance of ξ and η
$\mathbf{var}\ \xi$	Variance of ξ
$\mathcal{N}$	Set of all locally finite simple point patterns
$\mathfrak{N}$	Set of all locally finite counting measures
$\mathbf{1}$	Indicator function
$\text{Ber}(p)$	Bernoulli distribution with parameter p
$\text{Binom}(n, p)$	Binomial distribution with parameters n and p
$\text{Exp}(\lambda)$	Exponential distribution with parameter λ
$N(\mu, \sigma^2)$	Normal distribution with parameters μ and σ^2

$\mathrm{Pois}(\lambda)$	Poisson distribution with parameter λ
$\mathrm{Unif}(a, b)$	Uniform distribution on $[a, b]$

Graph Theory

$\mathbb{G}$	Graph
$\mathbb{V}$	Set of vertices
$\mathbb{E}$	Set of edges

Other Notations

E	Energy
V	Potential
$\mathtt{T}$	Tessellation
diag	Diagonal matrix
dist	Distance function
conv	Convex hull
$\lvert A\rvert$	Cardinality of a set A
$\lvert a\rvert$	Absolute value of a number a
$\lvert x\rvert$, $\lVert x\rVert_2$	Euclidean norm of a vector x
diam	Diameter
SO	Rotation group
$\mathrm{Im}\, z$	Imaginary part of a complex number
$\mathrm{Re}\, z$	Real part of a complex number
sgn	Signum function
Lip	Lipschitz operator
span	Linear hull
ν_d	Lebesgue measure
$V_0, \dots, V_d$	Intrinsic volumes
χ	Euler characteristic
K^r	r-neighbourhood of K
$\mathrm{Int}\, K$	Interior of K
$K\vert L$	Orthogonal projection of K onto L
o	The origin of $\mathbb{R}^d$
$B_r(o)$	Ball of radius r centred around the origin
T_x	Shift operator