

Editors:

J.-M. Morel, Cachan

B. Teissier, Paris

For further volumes:

<http://www.springer.com/series/304>



**FONDAZIONE
CIME**
ROBERTO CONTI

CENTRO INTERNAZIONALE MATEMATICO ESTIVO
INTERNATIONAL MATHEMATICAL SUMMER CENTER

Fondazione C.I.M.E., Firenze

C.I.M.E. stands for *Centro Internazionale Matematico Estivo*, that is, International Mathematical Summer Centre. Conceived in the early fifties, it was born in 1954 in Florence, Italy, and welcomed by the world mathematical community: it continues successfully, year for year, to this day.

Many mathematicians from all over the world have been involved in a way or another in C.I.M.E.'s activities over the years. The main purpose and mode of functioning of the Centre may be summarised as follows: every year, during the summer, sessions on different themes from pure and applied mathematics are offered by application to mathematicians from all countries. A Session is generally based on three or four main courses given by specialists of international renown, plus a certain number of seminars, and is held in an attractive rural location in Italy.

The aim of a C.I.M.E. session is to bring to the attention of younger researchers the origins, development, and perspectives of some very active branch of mathematical research. The topics of the courses are generally of international resonance. The full immersion atmosphere of the courses and the daily exchange among participants are thus an initiation to international collaboration in mathematical research.

C.I.M.E. Director

Pietro ZECCA

Dipartimento di Energetica "S. Stecco"

Università di Firenze

Via S. Marta, 3

50139 Florence

Italy

e-mail: zecca@unifi.it

C.I.M.E. Secretary

Elvira MASCOLO

Dipartimento di Matematica "U. Dini"

Università di Firenze

viale G.B. Morgagni 67/A

50134 Florence

Italy

e-mail: mascolo@math.unifi.it

For more information see CIME's homepage: <http://www.cime.unifi.it>

CIME activity is carried out with the collaboration and financial support of:

- INdAM (Istituto Nazionale di Alta Matematica)
- MIUR (Ministero dell'Università e della Ricerca)

Anna Capietto • Peter Kloeden • Jean Mawhin
Sylvia Novo • Rafael Ortega

Stability and Bifurcation Theory for Non-Autonomous Differential Equations

Cetraro, Italy 2011

Editors:
Russell Johnson
Maria Patrizia Pera

 Springer



Anna Capietto
Università di Torino
Dipartimento di Matematica
Torino, Italy

Sylvia Novo
E. de Ingenierías Industriales
Departamento de Matemática Aplicada
Universidad de Valladolid
Valladolid, Spain

Peter Kloeden
Universität Frankfurt
Institut für Mathematik
Frankfurt, Germany

Rafael Ortega
Universidad de Granada
Departamento de Matemática Aplicada
Granada, Spain

Jean Mawhin
Université Catholique de Louvain
Institut de Recherche
en Mathématique et Physique
Louvain-la-Neuve, Belgium

ISBN 978-3-642-32905-0 ISBN 978-3-642-32906-7 (eBook)
DOI 10.1007/978-3-642-32906-7
Springer Heidelberg New York Dordrecht London

Lecture Notes in Mathematics ISSN print edition: 0075-8434
ISSN electronic edition: 1617-9692

Library of Congress Control Number: 2012951625

Mathematics Subject Classification (2010): 34B15, 37B55, 34C25, 37E40, 37G35, 34K12

© Springer-Verlag Berlin Heidelberg 2013

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

The CIME session “Stability and Bifurcation Theory for Non-Autonomous Differential Equations” was held at Cetraro Italy, from 19 to 25 June 2011. This volume contains the notes of the five lecture courses which were held on that occasion.

One of our goals in organizing the session was to foster a comparison between the “topological” and “dynamical” approaches to the study of nonautonomous differential equations. Another goal was to facilitate the interaction between specialists versed in the one approach or the other. We are amply convinced that those goals were fulfilled.

In these course notes, the reader will find a systematic introduction to many of the themes and methods which make up the modern theory of nonautonomous differential and dynamical systems. Topics pertaining to differential equations in finite and infinite dimensions receive sustained attention. Also, discrete equations and systems have an important place in the notes. This is natural both because a differential equation is often studied via an appropriate discretization process and because nonautonomous discrete systems are of fundamental importance in their own right.

Here is a partial list of the various themes which were taken up in the course of the lectures: bounded orbits and stability in non-periodic monotone twist maps; properties of the minimal subsets of nonautonomous monotone differential-delay systems; resonance phenomena in nonautonomous ordinary differential equations; existence and properties of pullback attractors in skew-product dynamical systems; and the use of the Maslov index in bifurcation problems regarding nonautonomous Hamiltonian systems. Of course an impressive range of other topics was considered as well, and an ample quantity of specific problems was discussed.

The methods introduced by the speakers in the theoretical developments and in the treatment of specific problems may be divided into two classes. First, the use of “classical” techniques drawn from the topological degree theory, the calculus of variations, the search for upper/lower solutions, and others of a similar vein. Second, the use of “dynamical” constructs such as processes and skew-product flows, minimal sets and omega-limit sets, pullback attractors, invariant measures, and the like.

Here is a brief description of each of the courses which made up the session.

- Anna Capietto (Torino) considered a broad class of boundary value problems posed for nonautonomous nonlinear Hamiltonian systems. She stated and proved bifurcation results of “Rabinowitz” type for these problems. She showed how the Maslov index can be used as an effective tool in deriving such results.
- Peter Kloeden (Frankfurt) gave an introduction to the language and concepts of nonautonomous discrete dynamical systems. He discussed the theory of pullback attractors and went on to mention some results from nonautonomous bifurcation theory. He also took up some questions in the area of random dynamical systems.
- Jean Mawhin (Louvain-la-Neuve) discussed a number of illustrative nonlinear nonautonomous resonance problems. He effectively used a mix of methods drawn from the Leray–Schauder theory, the calculus of variations, and the technique of upper and lower solutions. He presented many results regarding existence and multiplicity of periodic solutions of certain paradigmatic periodically forced ODEs.
- Sylvia Novo (Valladolid) considered a significant class of nonautonomous functional differential equations having monotonicity properties. She studied the existence and the stability properties of minimal sets, together with the existence and structural properties of global attractors. She gave several applications, e.g., to the theory of neural networks and to that of compartmental systems.
- Rafael Ortega (Granada) first discussed the existence of bounded orbits and invariant curves for exact symplectic twist maps on the cylinder and especially on the plane. The results on invariant curves have stability statements as corollaries. He made use of a variational principle of Mather type. He then analyzed certain impact problems, especially the so-called ping-pong model.

The session was attended by about 50 scientists of “topological” and “dynamical” extractions. Their good-natured and active participation in the courses and their individual discussions helped to create a positive atmosphere which certainly facilitated the exchange of scientific ideas. We believe that the interaction between specialists in the topological and in the dynamical approaches to nonautonomous differential equations was greatly enriched by this CIME session.

Firenze, Italy
19 Dec 2011

Russell Johnson
Maria Patrizia Pera

Contents

The Maslov Index and Global Bifurcation for Nonlinear Boundary Value Problems	1
Alberto Boscaggin, Anna Capietto, and Walter Dambrosio	
1 Introduction and Classical Results	1
2 The Maslov Index	7
3 The Number of Moments of Verticality	12
4 The Phase-Angles and the Number of Moments of Verticality	16
5 Some Related Notions	21
6 Nonlinear First Order Systems in \mathbb{R}^{2N}	24
7 Nonlinear Dirac-Type Systems in the Half-Line.....	27
References	32
Discrete-Time Nonautonomous Dynamical Systems	35
P.E. Kloeden, C. Pötzsche, and M. Rasmussen	
1 Introduction	36
2 Autonomous Difference Equations	37
2.1 Autonomous Semidynamical Systems	39
2.2 Lyapunov Functions for Autonomous Attractors	40
3 Nonautonomous Difference Equations	43
3.1 Processes	44
3.2 Skew-Product Systems	45
4 Attractors of Processes	50
4.1 Nonautonomous Invariant Sets	51
4.2 Forwards and Pullback Convergence.....	52
4.3 Forwards and Pullback Attractors	53
4.4 Existence of Pullback Attractors	54
4.5 Limitations of Pullback Attractors	59
5 Attractors of Skew-Product Systems	61
5.1 Existence of Pullback Attractors	61
5.2 Comparison of Nonautonomous Attractors	64

5.3	Limitations of Pullback Attractors Revisited	66
5.4	Local Pullback Attractors	68
6	Lyapunov Functions	69
6.1	Existence of a Pullback Absorbing Neighbourhood System	70
6.2	Necessary and Sufficient Conditions	72
7	Bifurcations	79
7.1	Hyperbolicity and Simple Examples	80
7.2	Attractor Bifurcation	86
7.3	Solution Bifurcation	88
8	Random Dynamical Systems	92
8.1	Random Difference Equations	93
8.2	Random Attractors	94
8.3	Random Markov Chains	96
8.4	Approximating Invariant Measures	98
	References	100

Resonance Problems for Some Non-autonomous Ordinary

Differential Equations	103
-------------------------------------	-----

Jean Mawhin

1	Introduction, Notations and Preliminary Results	103
1.1	Introduction	103
1.2	Notations	105
1.3	Classes of Homeomorphisms	107
1.4	A Nonlinear Projector	109
2	Topological Approach	112
2.1	Introduction	112
2.2	Dirichlet Problem	112
2.3	Periodic Problem	114
2.4	Neumann Problem for Singular ϕ	145
3	Lagrangian Variational Approach	147
3.1	Introduction	147
3.2	The Functional Framework	147
3.3	Ground State Solutions	153
3.4	Saddle Point Solutions for Bounded Nonlinearities	159
3.5	Multiple Solutions Near Resonance	162
3.6	BV Periodic Solutions of the Forced Pendulum with Curvature Operator	169
4	Hamiltonian Variational Approach	172
4.1	Introduction	172
4.2	An Equivalent Hamiltonian System and Its Action	174
4.3	Multiplicity of Periodic Solutions	177
	References	179

Non-autonomous Functional Differential Equations and Applications	185
Sylvia Novo and Rafael Obaya	
1 Introduction	185
2 Basic Notions and Results	189
2.1 Flows Over Compact Metric Spaces	189
2.2 Almost Periodic and Almost Automorphic Dynamics	192
2.3 Some Important ODEs Examples	195
2.4 Ordered Banach Spaces: Monotone Skew-Product Semiflows	203
3 Non-autonomous FDEs with Finite Delay	206
3.1 Cooperative and Irreducible Systems of Finite Delay Equations	210
3.2 Semicontinuous Equilibria and Almost Automorphic Extensions	216
3.3 Monotone and Concave or Sublinear Cases	220
3.4 A Non-autonomous Cyclic Feedback System	225
3.5 Cellular Neural Networks	229
4 Non-autonomous Cyclic FDEs with Infinite Delay	237
4.1 Stability and Extensibility Results for Omega-Limit Sets	237
4.2 FDEs with Infinite Delay	240
4.3 Monotone FDEs with Infinite Delay	249
4.4 Compartmental Systems	255
References	259
Twist Mappings with Non-Periodic Angles	265
Markus Kunze and Rafael Ortega	
1 Introduction	265
2 Symplectic Maps in the Plane and in the Cylinder	270
3 The Twist Condition and the Generating Function	274
4 The Variational Principle	277
4.1 The Frenkel–Kontorowa Model	277
4.2 A General Framework	278
5 Existence of Complete Orbits	281
6 The Action Functional of a Newtonian Equation	286
7 Impact Problems and Generating Functions	290
7.1 The Dirichlet Problem	291
7.2 The Condition of Elastic Bouncing	293
7.3 A Bouncing Ball	294
8 Comments and Bibliographical Remarks	296
References	299
List of Participants	301