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Stability and Bifurcation Theory for Non-Autonomous Differential Equations

Cetraro, Italy 2011

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Preface

The CIME session "Stability and Bifurcation Theory for Non-Autonomous Differential Equations" was held at Cetraro Italy, from 19 to 25 June 2011. This volume contains the notes of the five lecture courses which were held on that occasion.

One of our goals in organizing the session was to foster a comparison between the "topological" and "dynamical" approaches to the study of nonautonomous differential equations. Another goal was to facilitate the interaction between specialists versed in the one approach or the other. We are amply convinced that those goals were fulfilled.

In these course notes, the reader will find a systematic introduction to many of the themes and methods which make up the modern theory of nonautonomous differential and dynamical systems. Topics pertaining to differential equations in finite and infinite dimensions receive sustained attention. Also, discrete equations and systems have an important place in the notes. This is natural both because a differential equation is often studied via an appropriate discretization process and because nonautonomous discrete systems are of fundamental importance in their own right.

Here is a partial list of the various themes which were taken up in the course of the lectures: bounded orbits and stability in non-periodic monotone twist maps; properties of the minimal subsets of nonautonomous monotone differential-delay systems; resonance phenomena in nonautonomous ordinary differential equations; existence and properties of pullback attractors in skew-product dynamical systems; and the use of the Maslov index in bifurcation problems regarding nonautonomous Hamiltonian systems. Of course an impressive range of other topics was considered as well, and an ample quantity of specific problems was discussed.

The methods introduced by the speakers in the theoretical developments and in the treatment of specific problems may be divided into two classes. First, the use of "classical" techniques drawn from the topological degree theory, the calculus of variations, the search for upper/lower solutions, and others of a similar vein. Second, the use of "dynamical" constructs such as processes and skew-product flows, minimal sets and omega-limit sets, pullback attractors, invariant measures, and the like.

Here is a brief description of each of the courses which made up the session.

- Anna Capietto (Torino) considered a broad class of boundary value problems posed for nonautonomous nonlinear Hamiltonian systems. She stated and proved bifurcation results of "Rabinowitz" type for these problems. She showed how the Maslov index can be used as an effective tool in deriving such results.
- Peter Kloeden (Frankfurt) gave an introduction to the language and concepts of nonautonomous discrete dynamical systems. He discussed the theory of pullback attractors and went on to mention some results from nonautonomous bifurcation theory. He also took up some questions in the area of random dynamical systems.
- Jean Mawhin (Louvain-la-Neuve) discussed a number of illustrative nonlinear nonautonomous resonance problems. He effectively used a mix of methods drawn from the Leray–Schauder theory, the calculus of variations, and the technique of upper and lower solutions. He presented many results regarding existence and multiplicity of periodic solutions of certain paradigmatic periodically forced ODEs.
- Sylvia Novo (Valladolid) considered a significant class of nonautonomous functional differential equations having monotonicity properties. She studied the existence and the stability properties of minimal sets, together with the existence and structural properties of global attractors. She gave several applications, e.g., to the theory of neural networks and to that of compartmental systems.
- Rafael Ortega (Granada) first discussed the existence of bounded orbits and invariant curves for exact symplectic twist maps on the cylinder and especially on the plane. The results on invariant curves have stability statements as corollaries. He made use of a variational principle of Mather type. He then analyzed certain impact problems, especially the so-called ping-pong model.

The session was attended by about 50 scientists of "topological" and "dynamical" extractions. Their good-natured and active participation in the courses and their individual discussions helped to create a positive atmosphere which certainly facilitated the exchange of scientific ideas. We believe that the interaction between specialists in the topological and in the dynamical approaches to nonautonomous differential equations was greatly enriched by this CIME session.

Firenze, Italy
19 Dec 2011

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Maria Patrizia Pera

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