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Irina Mitrea • Marius Mitrea

Multi-Layer Potentials and Boundary Problems

for Higher-Order Elliptic Systems in Lipschitz
Domains

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Preface

Layer potentials constitute one of the most powerful tools in the treatment of boundary value problems associated with (elliptic and parabolic) partial differential equations (PDEs). They have been traditionally employed in the context of second-order PDE, and one of the main goals of this monograph is to systematically develop a multilayer theory applicable to the higher-order setting. This extension of the classical theory is carried out in the context of arbitrary Lipschitz domains and includes mapping properties of such multilayers associated with complex, matrix-valued, constant coefficient, homogeneous elliptic systems, in function spaces suitably adapted to the higher-regularity case (of Besov and Triebel–Lizorkin type), Carleson measure estimates, non-tangential maximal function estimates, jump-relations, etc., which turn out to be just as versatile and effective as their second-order counterparts. In particular, this theory applies to such basic differential operators like the Laplacian, the bi-Laplacian, the polyharmonic operator, and the Lamé system of elasticity, though the gist of the present work is constructing, for the first time, a comprehensive theory (of Calderón–Zygmund type) for singular integral operators of multilayer type associated with generic higher-order PDEs and to discuss some of the implications of this multilayer theory to the well-posedness of boundary value problems for higher-order PDEs. As such, one of the main purposes of this monograph is to address an obvious gap/discrepancy/imbalance in the present literature between the second- and higher-order case.

The intended audience consists of any mathematically trained scientist with an interest in boundary value problems and partial differential equations. While this is an original research monograph, significant effort has been put in to make the material as reasonably accessible as possible. In particular, this monograph should also be useful to junior scientists working in the area of PDE.

Philadelphia, PA
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Contents

1	Introduction	1
2	Smoothness Scales and Calderón–Zygmund Theory in the Scalar-Valued Case	21
2.1	Lipschitz Domains and Non-tangential Maximal Function	21
2.2	Sobolev Spaces on Lipschitz Boundaries	31
2.3	Brief Review of Smoothness Spaces in \mathbb{R}^n	42
2.4	Smoothness Spaces in Lipschitz Domains	48
2.5	Weighted Sobolev Spaces in Lipschitz Domains	61
2.6	Stein’s Extension Operator on Weighted Sobolev Spaces in Lipschitz Domains	73
2.7	Other Smoothness Spaces on Lipschitz Boundaries	88
2.8	Calderón–Zygmund Theory in the Scalar-Valued Case	108
3	Function Spaces of Whitney Arrays	125
3.1	Whitney–Lebesgue and Whitney–Sobolev Spaces	125
3.2	Whitney–Besov Spaces	138
3.3	Multi-Trace Theory	142
3.4	Whitney–Hardy and Whitney–Triebel–Lizorkin Spaces	169
3.5	Interpolation	180
3.6	Whitney–BMO and Whitney–VMO Spaces	191
4	The Double Multi-Layer Potential Operator	199
4.1	Differential Operators and Fundamental Solutions	199
4.2	The Definition of Double Multi-Layer and Non-tangential Maximal Estimates	207
4.3	Carleson Measure Estimates	220
4.4	Jump Relations	233
4.5	Estimates on Besov, Triebel–Lizorkin, and Weighted Sobolev Spaces	245

5	The Single Multi-Layer Potential Operator	253
5.1	The Definition of Single Multi-Layer and Non-tangential Maximal Estimates.....	253
5.2	Carleson Measure Estimates.....	260
5.3	Estimates on Besov, Triebel–Lizorkin, and Weighted Sobolev Spaces.....	262
5.4	The Conormal Derivative	269
5.5	Jump Relations for the Conormal Derivative	284
6	Functional Analytic Properties of Multi-Layer Potentials and BVPs	293
6.1	Fredholm Properties of Boundary Multi-Layer Potentials	293
6.2	Compactness Criteria for the Double Multi-Layer on Whitney–Besov Spaces.....	327
6.3	Uniqueness for the Dirichlet Problem with Data in Whitney–Lebesgue Spaces	333
6.4	Boundary Problems on Besov and Triebel–Lizorkin Spaces	341
6.5	Boundary Problems for the Bi-Laplacian in Higher Dimensions	355
	References	405
	Symbol Index	411
	Subject Index	415
	Theorem Index	419
	Author Index	423