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Bo'az Klartag • Shahar Mendelson  
Vitali D. Milman  
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# Geometric Aspects of Functional Analysis

Israel Seminar 2006–2010

 Springer

**GFA**  
2006–2010

*Editors*

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# Preface

Since the mid-1980s the following volumes containing collections of papers reflecting the activity of the Israel Seminar in Geometric Aspects of Functional Analysis appeared:

- 1983–1984    Published privately by Tel Aviv University
- 1985–1986    Springer Lecture Notes in Mathematics, vol. 1267
- 1986–1987    Springer Lecture Notes in Mathematics, vol. 1317
- 1987–1988    Springer Lecture Notes in Mathematics, vol. 1376
- 1989–1990    Springer Lecture Notes in Mathematics, vol. 1469
- 1992–1994    Operator Theory: Advances and Applications, vol. 77, Birkhäuser
- 1994–1996    MSRI Publications, vol. 34, Cambridge University Press
- 1996–2000    Springer Lecture Notes in Mathematics, vol. 1745
- 2001–2002    Springer Lecture Notes in Mathematics, vol. 1807
- 2002–2003    Springer Lecture Notes in Mathematics, vol. 1850
- 2004–2005    Springer Lecture Notes in Mathematics, vol. 1910

The first six were edited by Lindenstrauss and Milman, the seventh by Ball and Milman and the last four by Milman and Schechtman.

As in the previous volumes, the current one reflects the general trends of the Theory. Most of the papers deal with different aspects of Asymptotic Geometric Analysis understood in a broad sense. It includes classical topics in the geometry of convex bodies, inequalities involving volumes of such bodies or, more generally, log-concave measures, valuation theory, probabilistic and isoperimetric problems in combinatorial setting. A special attention is given to the study of volume distribution on high dimensional spaces. Additional direction is the characterization of some classical constructions in Geometry and Analysis (like the Legendre and Fourier transforms, derivation and others) is represented by a few papers. This leads also to an unexpected use of fractional linear maps and one paper intensively study these maps and present their use in the Convexity Theory. In many of the papers Probability Theory plays an important role and probabilistic tools are used intensively. There are also papers on related subjects. All the papers here are original research papers and were subject to the usual standards of refereeing.

As in previous proceedings of the GAFA Seminar, we also list all the talks given in the seminar as well as talks in some related workshops and conferences. We believe this gives a sense of the main directions of research in our area.

We are grateful to Miriam Hercberg for taking excellent care of the typesetting aspects of this volume.

Tel Aviv, Israel  
Haifa, Israel  
Tel Aviv, Israel

Bo'az Klartag  
Shahar Mendelson  
Vitali Milman

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