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Vincent Rivasseau • Robert Seiringer
Jan Philip Solovej • Thomas Spencer

Quantum Many Body Systems

Cetraro, Italy 2010

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Preface

The idea that matter is composed by a huge number of particles (atoms) obeying simple laws of motion and that the macroscopic properties of bodies emerge as collective phenomena starting from this simple description dates back to Democritus (460–370 BC). Since then, physicists of all times have struggled with this fascinating idea in order to really understand the variety of our world starting from a simple microscopic description. Quantum many-body theory is nowadays the area of physics that provides the framework for understanding the collective behavior of vast assemblies of interacting quantum particles. Indeed, we know that matter is composed of atoms and molecules, interacting in accord with the laws of quantum mechanics. The equation describing the evolution and behavior of such assemblies, the Schrödinger equation with Coulomb forces, is well known: it can be written down and studied, and the properties of the system can in principle be understood at a qualitative and quantitative level, starting from this fundamental law. However, the number of variables required for describing the behavior of the microscopic constituents of any macroscopic body is enormous (it is of the order of the number of particles, which is $\sim 10^{23}$ for a mole of substance). Therefore, except for a few special but very important cases (e.g., ideal gases), the deduction of the macroscopic properties from the fundamental microscopic equation is a formidable mathematical challenge. This is, after all, not surprising: we know from experiments that complex and unexpected phenomena like superconductivity, superfluidity, and Bose–Einstein condensation, together with more familiar phenomena like magnetism or metallic behavior, emerge from the collective behavior of a huge number of quantum particles. These remarkable phenomena are a macroscopic manifestation of the law of quantum mechanics and cannot be understood by using Newtonian physics.

From the point of view of mathematical physicists, many-body theory provides an almost ideal field; the basic equation is known and one "just" needs to solve it in order to deduce and understand a number of interesting phenomena. Indeed, if the solutions of the Schrödinger equation with a large number of variables were known this would clearly be of extraordinary importance for physical applications as well as for technological developments. However, the mathematical difficulty of finding such solutions is enormous, but in past years a number of increasingly powerful and

sophisticated techniques have been developed to extract relevant information without having to solve the equation itself. These methods include multiscale analysis, functional inequalities, localization estimates, cluster expansions, supersymmetry and stochastic and conformal geometry. The beautiful mathematics related to these developments has also proved to be useful as a bridge between different disciplines, ranging from algebraic geometry to the study of the shape of bird flocks. The interaction of mathematical and physical complexity has proved to be very fruitful for both fields.

The aim of the *CIME school* on *Quantum Many-Body Systems*, which took place in Cetraro (Italy) from August 30 to September 4, 2010, was to provide an introduction to the beautiful and powerful mathematical techniques developed in this field. The school was attended by 30 participants from several different countries, including Austria, Denmark, France, Germany, Italy, the UK, Ukraine and the USA. Although the school was primarily intended for graduate students, the interesting topics and the high reputation of the lecturers also attracted several more senior researchers.

The school consisted of four series of lectures, presented by V. Rivasseau, R. Seiringer, J.P. Solovej, and T. Spencer, and each series was organized into four lessons of 2 h each (two in the morning and one in the afternoon). In addition, one afternoon was devoted to short presentations of the research activity of the younger participants and one evening to a short description of the activities of some of the senior participants.

The lectures of Prof. Rivasseau gave an introduction to some results in solid-state physics, obtained via constructive Renormalization Group methods. While the focus was on the proof of Fermi liquid behavior for a system of nonrelativistic two-dimensional fermions above the BCS transition temperature one lecture was devoted to the exciting perspectives opened by the use of the same methods to quantum gravity.

The lectures of Prof. Seiringer were devoted to the mathematical physics of Bose gases and Bose–Einstein condensation and included a rigorous proof of the latter for dilute, interacting gases in the so-called Gross–Pitaevski limit. Starting from the basic notions he also discussed advanced topics like the analysis of rotating traps and the emergence of lattices of quantized vortices.

Prof. Solovej provided a comprehensive introduction to quantum Coulomb systems. He gave a self-contained presentation of the functional analytic methods used to prove thermodynamic stability of coulombic matter, following a recently developed approach that allows to treat, on the same footing, translation and non-translation invariant systems of charged fermions and bosons.

Finally, Prof. Spencer described the rigorous and powerful methods of supersymmetry and their application to the problem of the localization–delocalization transition in the Anderson model and in random matrices. Moreover, he gave a tutorial review of some classical results and techniques, such as the use of Ward Identities in the XY model.

The atmosphere at the school was very lively, many questions and comments arose during and after each lecture, and scientific discussions took place; the

students profited very much from the possibility of close interactions with the lecturers.

As Editors of these Lectures Notes we would like to thank the people and institutions who contributed to the success of the course. In particular, it is our pleasure to thank the Scientific Committee of CIME for supporting our project; the Director, Prof. Pietro Zecca, and the Secretary, Prof. Elvira Mascolo, for their support during the organization. Special thanks go to the lecturers, who offered a unique occasion to the participants to enter this beautiful field.

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