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Spherical Harmonics and Approximations on the Unit Sphere: An Introduction

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Preface

Spherical harmonics have been studied extensively and applied to solving a wide range of problems in the sciences and engineering. Interest in approximations and numerical methods for problems over spheres has grown steadily. These notes provide an introduction to the theory of spherical harmonics in an arbitrary dimension as well as a summarizing account of classical and recent results on some aspects of approximation by spherical polynomials and numerical integration over the sphere. The notes are intended for graduate students in the mathematical sciences and researchers who are interested in solving problems involving partial differential and integral equations on the sphere, especially on the unit sphere $\mathbb{S}^2$ in $\mathbb{R}^3$. We also discuss briefly some related work for approximation on the unit disk in $\mathbb{R}^2$, with those results being generalizable to the unit ball in more dimensions. The subject of theoretical approximation of functions on $\mathbb{S}^d$, $d > 2$, using spherical polynomials has been an active area of research over the past several decades. We summarize some of the major results, giving some insight into them; however, these notes are not intended to be a complete development of the theory of approximation of functions on $\mathbb{S}^d$ by spherical polynomials.

There are a number of other approaches to the approximation of functions on the sphere. These include spline functions on the sphere, wavelets, and meshless discretization methods using radial basis functions. For a general survey of approximation methods on the sphere, see Fasshauer and Schumaker [46]; and for a more complete development, see Freeden et al. [47]. For more recent books devoted to radial basis function methods, see Buhmann [24], Fasshauer [45], and Wendland [118]. For a recent survey of numerical integration over $\mathbb{S}^2$, see Hesse et al. [63].

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Contents