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Many mathematicians from all over the world have been involved in a way or another in C.I.M.E.'s activities over the years. The main purpose and mode of functioning of the Centre may be summarised as follows: every year, during the summer, sessions on different themes from pure and applied mathematics are offered by application to mathematicians from all countries. A Session is generally based on three or four main courses given by specialists of international renown, plus a certain number of seminars, and is held in an attractive rural location in Italy.

The aim of a C.I.M.E. session is to bring to the attention of younger researchers the origins, development, and perspectives of some very active branch of mathematical research. The topics of the courses are generally of international resonance. The full immersion atmosphere of the courses and the daily exchange among participants are thus an initiation to international collaboration in mathematical research.

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Multiscale and Adaptivity: Modeling, Numerics and Applications

C.I.M.E. Summer School,
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Preface

The CIME-EMS Summer School in applied mathematics on “Multiscale and Adaptivity: Modeling, Numerics and Applications” was held in Cetraro (Italy) from July 6 to 11, 2009. This course has focused on mathematical methods for systems that involve multiple length/time scales and multiple physics. The complexity of the structure of these systems requires suitable mathematical and computational tools. In addition, mathematics provides an effective approach toward devising computational strategies for handling multiple scales and multiple physics. This course brought together researchers and students from different areas such as partial differential equations (PDEs), analysis, mathematical physics, numerical analysis, and scientific computing to address the challenges present in these issues. Physical, chemical, and biological processes for many problems in computational physics, biology, and material science span length and time scales of many orders of magnitude. Traditionally, scientists and research groups have focused on methods that are particularly applicable in only one regime, and knowledge of the system at one scale has been transferred to another scale only indirectly. Microscopic models, for example, have been often used to find the effective parameters of macroscopic models, but for obvious computational reasons, microscopic and macroscopic scales have been treated separately.

The enormous increase in computational power available (due to the improvement both in computer speed and in efficiency of the numerical methods) allows in some cases the treatment of systems involving scales of different orders of magnitude, arising, for example, when effective parameters in a macroscopic model depend on a microscopic model, or when the presence of a singularity in the solution produces a continuum of length scales. However, the numerical solution of such problems by classical methods often leads to an inefficient use of the computational resources, even up to the point that the problem cannot be solved by direct numerical simulation. The main reasons for this are that the necessary resolution of a fine scale entails an over-resolution of coarser scales, the position of the singularity is not known beforehand, the gap between the scales is too big for a treatment in the same framework. In other cases, the structure of the mathematical models that treat the system at the different scales varies a lot, and therefore new mathematical techniques

are required to treat systems described by different mathematical models. Finally, in many cases one is interested in the accurate treatment of a small portion of a large system, and it is too expensive to treat the whole system at the required accuracy. In such cases, the region of interest is modeled and discretized with great accuracy, while the remaining parts of the system are described by some reduced model, which enormously simplifies the calculation, still providing reasonable boundary conditions for the region of interest, allowing the required level of detail in such region.

The outstanding and internationally renowned lecturers have themselves contributed in an essential way to the development of the theory and techniques that constituted the subjects of the courses. The selection of the five topics of the CIME-EMS Course was not an easy task because of the wide spectrum of recent developments in multiscale methods and models. The six world leading experts illustrated several aspects of the multiscale approach.

Silvia Bertoluzza, from IMATI-CNR Pavia, described the concept of nonlinear sparse wavelet approximation of a given (known) function. Next she showed how the tools just introduced can be applied in order to write down efficient adaptive schemes for the solution of PDEs.

Bjorn Engquist, from ICES University of Texas at Austin, gradually guided the audience toward the realm of "Multiscale Modeling," by providing mathematical ground for state-of-the-art analytical and numerical multiscale problems.

Alfio Quarteroni, from EPFL, Lausanne, and Politecnico di Milano, considered adaptivity in mathematical modeling for the description and simulation of complex physical phenomena. He showed that the combination of hierarchical mathematical models can be set up with the aim of reducing the computational complexity in the real life problems.

Ricardo H. Nochetto, from University of Maryland, and Andreas Veeser, from Università di Milano, in their joint course started with an overview of the a posteriori error estimation for finite element methods, and then they exposed recent results about the convergence and complexity of adaptive finite element methods.

Kunibert G. Siebert, from Universität Duisburg-Essen, described the implementation of adaptive finite element methods using toolbox ALBERTA (created by Alfred Schmidt and Kunibert G. Siebert, which is freely available).

The main "senior" lecturers were complemented by four young speakers, who gave account of detailed examples or applications during an afternoon session dedicated to them. Matteo Semplice, Università dell'Insubria, has spoken about "Numerical entropy production and adaptive schemes for conservation laws," Tiziano Passerini, from Emory University, about "A 3D/1D geometrical multiscale model of cerebral vasculature," Loredana Gaudio, MOX Politecnico di Milano, about "Spectral element discretization of optimal control problems," and Carina Geldhauser, Universität Tuebingen, described "A discrete-in-space scheme converging to an unperturbed Cahn–Hilliard equation." Both the lectures and the active interactions with and within the audience contributed to the scientific success of the course, which was attended by about 60 people of various nationality (14 different countries), ranging from first year PhD students to full professors. The present

volume collects the expanded version of the lecture notes by Silvia Bertoluzza, Alfio Quarteroni (with Marco Discacciati and Paola Gervasio as coauthors), Ricardo H. Nochetto, Andreas Veeser, and Kunibert G. Siebert. We are grateful to them for such high quality scientific material.

As editors of these Lecture Notes and as scientific directors of the course, we would like to thank the many persons and Institutions that contributed to the success of the school. It is our pleasure to thank the members of the Scientific Committee of CIME for their invitation to organize the School; the Director, Prof. Pietro Zecca, and the Secretary, Prof. Elvira Mascolo, for their efficient support during the organization and their generous help during the school. We were particularly pleased by the fact that the European Mathematical Society (EMS) chose to cosponsor this CIME course as one of its Summer School in applied mathematics for 2009. Our special thanks go to the lecturers for their early preparation of the material to be distributed to the participants, for their excellent performance in teaching the courses and their stimulating scientific contributions. All the participants contributed to the creation of an exceptionally friendly atmosphere in the beautiful environment around the School. We also wish to thank Dipartimento di Matematica of the Università degli Studi di Milano, and Dipartimento di Matematica ed Informatica of the Università degli Studi di Catania for their financial support.

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