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Spectral Analysis on Graph-Like Spaces

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Preface

In this monograph, we analyse thin tubular structures, so-called "graph-like spaces", and their natural limits, when the radius of a graph-like space tends to zero. The limit space is typically a metric graph, i.e. a graph, where each edge is associated a length, and therefore, the space turns into a one-dimensional manifold with singularities at the vertices. On both, the graph-like spaces and the metric graph, we can naturally define Laplace-like differential operators. We are interested in asymptotic properties of such operators. In particular, we show norm resolvent convergence, convergence of the spectra and resonances.

Tubular structures with small radius have attracted a lot of attention in the last years. Tubular structures are frequently used in different areas such as mathematical physics to describe properties of nano-structures, in spectral geometry to provide examples with given spectral properties, or in global analysis to calculate spectral invariants.

Since the underlying spaces in the thin radius limit change, and even become singular in the limit, we develop new tools such as

- Norm convergence of operators acting in different Hilbert spaces.
- An extension of the concept of boundary triples to partial differential operators.
- An abstract definition of resonances via boundary triples.

These tools are formulated in an abstract framework, independent of the original problem of graph-like spaces, and in a way that they may be applied in many other situations, for example when the underlying space is geometrically perturbed.

We briefly outline the content of this work. Chapter 1 is devoted to an exemplary overview of the results proven in this work, their history and a discussion of further research. In Chap. 2, we introduce the necessary concepts for discrete and metric graphs, especially Laplace-type operators. Chapter 3 contains partially new material, in particular, boundary triples associated with a quadratic form with applications to the PDE case. Moreover, we derive an abstract version of complex scaling via boundary triples and introduce resonances, i.e. poles of a meromorphic continuation of the resolvent. Chapter 4 provides new material on (norm-)convergence of operators and quadratic forms in different Hilbert spaces. We

extend these concepts to non-self-adjoint operators. Chapter 5 provides perturbation arguments for manifolds under a change of the Riemannian metric. Moreover, we present how a tubular neighbourhood can be reduced to the underlying one-dimensional space using separation of variables.

In Chap. 6, we define abstract graph-like manifolds associated with a star-shaped graph. We allow different scaling behaviours of the vertex neighbourhood and different boundary conditions, leading to different limit operators. Finally, in Chap. 7, we combine the convergence results for star-shaped graphs in order to get convergence results for general metric graphs and graph-like manifolds. We use the language of boundary triples in order to combine the local convergence results for star-graphs to convergence results for general graphs. Combining and extending existing results in the literature, we show norm-resolvent convergence of the corresponding Laplacians, convergence of the spectrum and of resonances.

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