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Distance Expanding Random Mappings, Thermodynamical Formalism, Gibbs Measures and Fractal Geometry

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Preface

In this book we introduce measurable expanding random systems, develop the thermodynamical formalism and establish, in particular, exponential decay of correlations and analyticity of the expected pressure although the spectral gap property does not hold. This theory is then used to investigate fractal properties of conformal random systems. We prove a Bowen's formula and develop the multifractal formalism of the Gibbs states. Depending on the behavior of the Birkhoff sums of the pressure function we get a natural classification of the systems into two classes: *quasi-deterministic* systems which share many properties of deterministic ones and *essential* random systems which are rather generic and never bi-Lipschitz equivalent to deterministic systems. We show in the essential case that the Hausdorff measure vanishes which refutes a conjecture of Bogenschütz and Ochs. We finally give applications of our results to various specific conformal random systems and positively answer a question of Brück and Bürger concerning the Hausdorff dimension of random Julia sets.

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