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Habib Ammari

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# Mathematical Modeling in Biomedical Imaging II

Optical, Ultrasound, and Opto-Acoustic  
Tomographies

 Springer

*Editor*  
Habib Ammari  
École Normale Supérieure  
Mathématiques et Applications  
45 rue d'Ulm  
75005 Paris  
France  
habib.ammari@ens.fr

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## Preface

The aim of this volume is to report recent mathematical and computational advances in optical, ultrasound, and photo-acoustic (also called opto-acoustic) tomographies. The volume outlines the state-of-the-art and future directions in optical and ultrasound imaging. It provides some of the most recent mathematical and computational tools in these fields. It is particularly suitable for researchers and graduate students in applied mathematics and in biomedical engineering.

Ultrasound imaging is based on the detection of mechanical properties (acoustic impedance) in biological soft tissues. It can provide good spatial resolution because of its millimetric wavelength and weak scattering at MHz frequencies. However, soft-tissue contrast is relatively poor. Optical tomography is a biomedical imaging modality that uses scattered light as a probe of structural variations in the optical properties of tissue. Optical imaging is very sensitive to optical absorption but can only provide a spatial resolution on the order of 1 cm at cm depths.

Photo-acoustic imaging is a promising new biomedical imaging modality. It combines both optical and ultrasound approaches to provide images of optical contrasts (based on the optical absorption) with ultrasonic resolution.

The objective of the volume is fourfold: (i) to discuss models for light propagation and present fast algorithms for solving the radiative transfer equation; (ii) to provide efficient weighted-migration algorithms for detecting acoustic anomalies and to investigate the coherent interferometric imaging strategy in the acoustic wave propagation regime relevant for biomedical applications; (iii) to explain some experimental setups and survey mathematical inversion techniques in photo-acoustic tomography; (iv) to compensate the effect of acoustic attenuation in purely acoustic as well as photo-acoustic imaging.

The book is organized as follows. Chapter 1 outlines recent mathematical advances in the image reconstruction problem of optical tomography. It gives models for light propagation on microscopic, mesoscopic and macroscopic scales. The mathematical formulation of the corresponding forward problem is dictated primarily by spatial scale, ranging from the Maxwell equations at the

microscale, to the radiative transport equation at the mesoscale, and to diffusion theory at the macroscale. The corresponding inverse problem that arises at each of these scales of reconstructing the optical properties of a medium of interest from boundary measurements is considered. An emphasis is put on direct methods for image reconstruction.

Chapter 2 reports recent mathematical and computational advances in the image reconstruction problem of ultrasound tomography. It discusses expansion methods and reverse migration algorithms for detecting acoustic anomalies. When the acoustic medium is randomly heterogeneous, travel times cannot be known with accuracy so that images obtained with reverse migration are noisy and not statistically stable, that is, they change with the realization of the random medium. Coherent interferometry (CINT) has been shown to achieve a good compromise between resolution and deblurring for imaging in noisy environments. CINT consists of backpropagating the cross correlations of the recorded signals over appropriate space-time or space-frequency windows rather than the signals themselves. Chapter 2 provides a CINT strategy in the acoustic wave propagation regime relevant for biomedical applications.

Chapter 3 is devoted to photo-acoustic tomography. Photo-acoustic tomography utilizes opto-acoustic effects of an absorbing medium; when a sample is illuminated by a short electromagnetic pulse, such as visible light, or radio wave, it induces an acoustic wave. The generated pressure field of the acoustic wave depends on the spatially varying absorption density of the sample.

In photo-acoustic imaging the goal is to recover the density function from measurement data of the acoustic pressure taken outside the illuminated sample. Chapter 3 outlines the principles of photo-acoustic tomography. It presents a few reconstruction algorithms which allow to correct the effects of imposed boundary conditions and of acoustic attenuation in photo-acoustic image reconstruction. In photo-acoustic imaging, if the medium is acoustically homogeneous and has the same acoustic properties as the free space, then the boundary of the object plays no role and the optical properties of the medium can be extracted from measurements of the pressure wave by inverting a spherical Radon transform. However, if a boundary condition has to be imposed on the pressure field, then there is no explicit inversion formula. Using a quite simple duality approach, one can still reconstruct the optical absorption coefficient.

Chapter 3 investigates quantitative photo-acoustic imaging in the case of a bounded medium with imposed boundary conditions. It proposes a geometric-control approach to deal with the case of limited view measurements. For small optical absorbers in a non-absorbing background, Chapter 3 provides adapted algorithms to identify the locations of the absorbers and estimate their absorbed energy. An efficient approach in the case of extended optical sources and attenuating acoustic background is also designed. By testing the boundary measurements against an appropriate family of functions, one can access the Radon transform of the initial condition, and thus recovers

quantitatively any initial condition for the photo-acoustic problem. Chapter 3 shows how to compensate the effect of acoustic attenuation on image quality for extended absorbers.

Chapter 4 goes further on the investigation of the effect of attenuation on photo-acoustic imaging. The existing attenuation models are reviewed in some detail and their causality is discussed; which is an essential property for algorithms for inversion with attenuated data. Then, it surveys causality properties of common attenuation models. Integro-differential equations which the attenuated waves are satisfying are derived. In addition Chapter 4 shows the ill-conditionness of the inverse problem for calculating the non-attenuated wave from the attenuated one.

Chapter 5 is devoted to quantitative photo-acoustic tomography. The problem reduces to reconstruct optical maps, particularly the absorption coefficient, from the deposited optical energy. To reconstruct different maps, there are three methodologies. First, using single optical illumination and assuming that the scattering map is known, one can recover the absorption map; second, using multi-wavelength illuminations and assuming the spectral model of optical coefficients, both absorption and scattering maps can be obtained; third, using multiple optical illumination both absorption and scattering maps can be recovered. Chapter 5 mainly focuses on the third strategy. It provides an efficient algorithm for large-scale three-dimensional quantitative photo-acoustic reconstructions to simultaneously reconstruct the absorption coefficient and scattering coefficient.

The partial support by the ANR project EchoScan (AN-06-Blan-0089) and the ERC Advanced Grant Project MULTIMOD-267184 are acknowledged. We also thank the staff at the Institute Henri Poincaré.

Paris

*Habib Ammari*





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