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Nonlinear PDE's and Applications

C.I.M.E. Summer School,
Cetraro, Italy 2008

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Preface

This volume collects the notes of the CIME course *Nonlinear PDE's and applications* held in Cetraro (Italy) on June 23–28, 2008. The school consisted in 5 series of lectures, delivered by

Stefano Bianchini (SISSA, Trieste)
Eric A. Carlen (Rutgers University)
Alexander Mielke (WIAS, Berlin)
Felix Otto (Bonn University)
Cedric Villani (Ecole Normale Superieure de Lyon).

They presented a broad overview on some deep results and new exciting developments concerning in particular optimal transport theory, nonlinear evolution equations, functional inequalities, and differential geometry. A brief (and largely incomplete) account of the main topics considered here involves optimal transport, Hamilton–Jacobi equations, Riemannian geometry, and their links with sharp geometric/functional inequalities, variational methods for studying nonlinear evolution equations and their scaling properties, the metric/energetic theory of gradient flows and of rate-independent evolution problems.

The course aimed at showing the deep connections among all these topics and at opening new research directions, through the contribution of leading experts in these fields.

Stefano Bianchini gave a course on

Transport rays and applications to Hamilton–Jacobi equations

showing new recent results and applications of geometric measure/disintegration theory to Hamilton–Jacobi equations and optimal transportation. The tools developed here lie at the core of new relevant achievements concerning the existence of transport maps for general Monge problems, leading eventually to a general solution of Monge's problem in Euclidean spaces, in the case cost=distance, when the distance is induced by an arbitrary norm.

The course of *Eric Carlen* on

Sharp functional inequalities and nonlinear evolution equations

discussed in a unified perspective the interplay between sharp functional inequalities (as the ones of Hardy-Littlewood-Sobolev, Onofri, Brascamb-Lieb) and the asymptotic behaviour of solutions to certain evolution equations. Many different techniques, involving rearrangement, symmetrization, entropy dissipation, gradient flows, and calculus of variations, enter as a crucial tool in the arguments and shed new light on this fascinating subject.

Alexander Mielke presented in his course

Differential, energetic and metric formulations for rate-independent processes

a general theory covering a wide spectrum of rate-independent problems arising in dry friction, elastoplasticity, magnetism, and phase transformation models. The notes cover many different approaches to this kind of evolutionary phenomena, starting from the "energetic" point of view, based on a recursive minimization of suitable quasi-static functionals driving the evolution. Different kind of solutions are then considered and the so called "viscous" approach leading to the recent notion of BV-solution has been investigated.

Felix Otto delivered a series of lectures on

Scaling laws by PDE methods

dealing with the problem of obtaining sharp asymptotic estimates on the evolution of suitable macroscopic quantities in some classes of nonlinear time-dependent problems. Even though the problems under consideration are quite different (Cahn–Hilliard equations, convection-diffusion Rayleigh-Bernard equations, Kuramoto-Sivashinsky evolution), in all cases lower bounds are obtained, which are sharp or almost sharp for generic solutions.

Cedric Villani gave a course on

Optimal transport and curvature

presenting the main results of optimal transport theory in Riemannian manifolds and their geometric counterparts. In particular, the link between lower bounds on Ricci curvature and the displacement convexity of integral/entropy functionals in the space of probability measures has been analyzed: it allows to prove stability of the lower Ricci curvature bounds with respect to measured Gromov-Hausdorff convergence and it leads to a synthetic formulation of lower Ricci bounds for metric measure spaces. The second part of the course has been devoted to the regularity theory of optimal transport maps, a theme now under active investigation, with deep links with the geometry of the ambient manifold: its cut-locus, its sectional curvature, and more.

This series of lectures attracted more than 80 participants, largely PhD students or post-docs, but also senior researchers; even though it is impossible to give a comprehensive account of the main themes of nonlinear PDE's, which cannot be exhausted in any kind of school or concentration period, we

believe that this CIME course has been rich of useful suggestions and ideas for inspiring new researches and developments in the near future.

We wish to thank all the lecturers for their active participation and their valuable contribution, Alessio Figalli, Matteo Gloyer, and Riccarda Rossi for their worthwhile assistance in preparing the present volume, and the CIME foundation, in particular the director Prof. Pietro Zecca and the secretary Prof. Elvira Mascolo, for their helpful support and for the organization of such a remarkable event in Cetraro.

November 26, 2009

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