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Damped Oscillations of Linear Systems

A Mathematical Introduction

Springer

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To my wife.

Foreword

The theory of linear damped oscillations has been studied for more than 100 years and is still of vital interest to researchers in Control Theory, Optimization, and computational aspects. This theory plays a central role in studying the stability of mechanical structures, but it has applications to other fields such as electrical network systems or quantum mechanical systems. We have purposely restricted ourselves to the basic model leaving aside gyroscopic effects and free rigid body motion. In contrast, the case of a singular mass matrix is analysed in some detail. We spend quite a good deal of time discussing underlying spectral theory, not forgetting to stress its limitations as a tool for our ultimate objective – the time behaviour of a damped system. We have restricted ourselves to finite dimension although we have attempted, whenever possible, to use methods which allow immediate generalisation to the infinite-dimensional case.

Our text is intended to be an introduction to this topic and so we have tried to make the exposition as self-contained as possible. This is also the reason why we have restricted ourselves to finite dimensional models. This lowering of the technical barrier should enable the students to concentrate on central features of the phenomenon of damped oscillation.

The introductory chapter includes some paragraphs on the mechanical model in order to accommodate readers with weak or no background in physics.

The text presents certain aspects of matrix theory which are contained in monographs and advanced textbooks but may not be familiar to the typical graduate student. One of them is spectral theory in indefinite product spaces. This topic receives substantial attention because it is a theoretical fundament for our model. We do not address numerical methods especially designed for this model. Instead we limit ourselves to mention what can be done with most common matrix algorithms and to systematically consider the sensitivity, that is, *condition number estimates and perturbation properties* of our topics. In our opinion numerical methods, in particular invariant-subspace reduction

for J-symmetric matrices are still lacking and we have tried to make a case for a more intensive research in this direction.

Our intention is to take readers on a fairly sweeping journey from the basics to several research frontiers. This has dictated a rather limited choice of material, once we ‘take off’ from the basics. The choice was, of course, closely connected with our own research interests. In some cases we present original research (cf. e.g. Chap. 19), whereas we sometimes merely describe an open problem worth investigating – and leave it unsolved.

This text contains several contributions of our own which may be new. As a rule, we did not give any particular credit for earlier findings due to other authors or ourselves except for more recent ones or when referring to further reading. Our bibliography is far from exhaustive, but it contains several works offering more detailed bibliographical coverage.

The text we present to the reader stems from a synonymous course I have taught for graduate and post-doc students of the Department of Mathematics, University of Osijek, Croatia in the summer term 2007/2008. It took place at the Department of Mathematics at the University of Osijek and was sponsored by the program ‘Brain gain visitor’ from the National Foundation for Science, Higher Education and Technological development of the Republic of Croatia. Due to its origin, it is primarily designed for students of Mathematics but it will be of use also to engineers with enough mathematical background. Even though the text does not cover all aspects of the linear theory of damped oscillations, I hope that it will also be of some help to the researchers in this field.

In spite of a good deal of editing the text still contains some remnants of its oral source. This pertains to the sometimes casual style more suited to a lecture than to a monograph – as was the original aim of this work.

Bibliographical Notes and Remarks are intended to broaden the scope by mentioning some other important directions of research present and past. According to our bibliographical policy, when presenting a topic we usually cite at most one or two related works and refer to their bibliographies.

It is a pleasure to thank the participants in the course, in particular professor N. Truhar and his collaborators for their discussions and support during the lectures in Osijek. The author was very fortunate to have obtained large lists of comments and corrections from a number of individuals who have read this text. These are (in alphabetic order): K. Burazin (Osijek), L. Grubišić (Zagreb), D. Kressner (Zürich), P. Lancaster, (Calgary), H. Langer (Vienna), I. Matić (Osijek), V. Mehrmann (Berlin), M. Miloloža Pandur (Osijek), B. Parlett (Berkeley), I. Nakić (Zagreb), A. Suhadolc (Ljubljana), I. Veselić (Chemnitz), A. Wiegner (Hagen) and, by no means the least, the anonymous referees. Many of them went into great detail and/or depth. This resulted in substantial revisions of the whole text as well as in significant enlargements. To all of them I am deeply obliged. Without their generous support and encouragement this text would hardly deserve to be presented to the public. The financial help of both National Foundation for

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Osijek/Hagen *K. Veselić*
July 2008–December 2010

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Introduction

Here is some advice to make this book easier to read.

In order to check/deepen the understanding of the material and to facilitate independent work the reader is supplied with some thoroughly worked-out examples as well as a number of exercises. Not all exercises have the same status. Some of them are 'obligatory' because they are quoted later. Such exercises are either easy and straightforward or accompanied by hints or sketches of solution. Some just continue the line of worked examples. On the other end of the scale there are some which introduce the reader to research along the lines of the development. These are marked by the word 'try' in their statement. It is our firm conviction that a student can fully digest these Notes only if he/she has solved a good quantity of exercises.

Besides examples and exercises there are a few theorems and corollaries the proof of which is left to the reader. The difference between a corollary without proof and an exercise without solution lies mainly in their significance in the later text.

We have tried to minimise the interdependencies of various parts of the text.

What can be skipped on first reading? Most of the exercises. Almost none of the examples. Typically the material towards the end of a chapter. In particular

- Chapter 10: Theorem 10.12
- Chapter 12: Theorem 12.17
- Any of Chaps. 19–22

Prerequisites and terminology. We require standard facts of matrix theory over the real or complex field Ξ including the following (cf. [31, 58]).

- Linear (in)dependence, dimension, orthogonality. Direct and orthogonal sums of subspaces.
- Linear systems, rank, matrices as linear maps. We will use the terms *injective/surjective* for a matrix with linearly independent columns/rows.

- Standard matrix norms, continuity and elementary analysis of matrix-valued functions.
- Standard matrix decompositions such as
  - Gaussian elimination and the LU-decomposition $A = LU$, L lower triangular with unit diagonal and U upper triangular.
  - Idem with pivoting $A = LU\Pi$, L, U as above and Π a permutation matrix, corresponding to the standard row pivoting.
  - Cholesky decomposition of a positive definite Hermitian matrix $A = LL^*$, L lower triangular.
  - QR-decomposition of an arbitrary matrix $A = QR$, Q unitary and R upper triangular.
  - Singular value decomposition of an arbitrary matrix $A = U\Sigma V^*$, U, V unitary and $\Sigma \geq 0$ diagonal.
  - Eigenvalue decomposition of a Hermitian matrix $A = U\Lambda U^*$, U unitary, Λ diagonal.
  - Simultaneous diagonalisation of a Hermitian matrix pair A, B (the latter positive definite)

$$\Phi^* A \Phi = \Lambda, \quad \Phi^* B \Phi = I, \quad \Lambda \text{ diagonal.}$$

  - Schur (or triangular) decomposition of an arbitrary square $A = UTU^*$, U unitary, T upper triangular.
  - Polar decomposition of an arbitrary $m \times n$-matrix with $m \geq n$

$$A = U\sqrt{A^* A} \text{ with } U^* U = I_n.$$

- Characteristic polynomial, eigenvalues and eigenvectors of a general matrix; their geometric and algebraic multiplicity.
- (Desirable but not necessary) Jordan canonical form of a general matrix.

Further general prerequisites are standard analysis in Ξ^n as well as elements of the theory of analytic functions.

Notations. Some notations were implicitly introduced in the lines above. The following notational rules are not absolute, that is, exceptions will be made, if the context requires them.

- *Scalar:* Lower case Greek $\alpha, \lambda, \ldots$.
- *Column vector:* Lower case Latin $x, y, \ldots$, their components: $x_j, y_j, \ldots$ (or $(x)_j, (y)_j, \ldots$)
- *Canonical basis vectors:* e_j as $(e_j)_k = \delta_{jk}$.
- *Matrix order:* $m \times n$, that is, the matrix has m rows and n columns, if a matrix is square then we also say it to be of order n.
- The set of all $m \times n$-matrices over the field Ξ: $\Xi^{m,n}$

- *Matrix:* Capital Latin $A, B, X, \ldots$, sometimes capital Greek $\Phi, \Psi, \ldots$; diagonal matrices: Greek $\boldsymbol{\alpha}, \Lambda, \ldots$ (bold face, if lower case). By default the order of a general square matrix will be n, for phase-space matrices governing damped systems this order will mostly be $2n$.
- *Identity matrix, zero matrix:* I_n, $0_{m,n}$, 0_n; the subscripts will be omitted whenever clear from context.
- *Matrix element:* Corresponding lower case of the matrix symbol $A = (a_{ij})$.
- *Block matrix:* $A = (A_{ij})$.
- *Matrix element in complicated expressions:* $ABC = ((ABC)_{ij})$.
- *Diagonal matrix:* $\mathrm{diag}(a_1, \ldots, a_n)$;block *Diagonal matrix:*$\mathrm{diag}(A_1, \ldots, A_p)$.
- *Matrix transpose:* $A^T = (a_{ji})$.
- *Matrix adjoint:* $A^* = (\overline{a_{ji}})$.
- *Matrix inverse and transpose/adjoint:* $(A^T)^{-1} = A^{-T}, \quad (A^*)^{-1} = A^{-*}$.
- *The null space and the range of a matrix as a linear map:* $\mathcal{N}(A)$, $\mathcal{R}(A)$.
- *Spectrum:* $\sigma(A)$.
- *Spectral radius:* $\mathrm{spr}(A) = \max|\sigma(A)|$.
- *Spectral norm:* $\|A\| = \sqrt{\mathrm{spr}(A^*A)}$ and condition number $\kappa(A) = \|A\|\|A^{-1}\|$.
- *Euclidian norm:* $\|A\|_E = \sqrt{\mathrm{Tr}(A^*A)}$ and condition number $\kappa_E(A) = \|A\|_E\|A^{-1}\|_E$. In the literature this norm is sometimes called the Frobenius norm or the Hilbert-Schmidt norm.
- Although a bit confusing the term *eigenvalues* (in plural) will mean any sequence of the complex zeros of the characteristic polynomial, counted with their multiplicity. Eigenvalues as elements of the spectrum will be called *spectral points* or *distinct eigenvalues*.