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Disorder and Critical Phenomena Through Basic Probability Models

École d'Été de Probabilités de Saint-Flour XL – 2010

Springer

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ISBN 978-3-642-21155-3 e-ISBN 978-3-642-21156-0
DOI 10.1007/978-3-642-21156-0
Springer Heidelberg Dordrecht London New York

Lecture Notes in Mathematics ISSN print edition: 0075-8434
ISSN electronic edition: 1617-9692

Library of Congress Control Number: "PCN applied for"

Mathematics Subject Classification (2011): 82B44, 60K35, 60K37, 82B27, 60K05, 82D30

Cover design: deblik, Berlin

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

These notes are a revised version of the ones that I have prepared and used for my course at the 40th Saint-Flour Probability Summer School. I was extremely happy to receive the invitation and giving the course has been a real pleasure. This marks my my third time participating in the school and this preface is the occasion to compliment Jean Picard for his discrete, smooth and efficient way of running it.

This invitation gave me the opportunity to rethink my research activities in the last 5 or 6 years, trying to take a somewhat different standpoint: in the end I realized that I was just trying to go back to the original motivations. In this period in fact I have been working mostly on localization phenomena in certain disordered systems and a class of models – the pinning models – took a leading position. But the question driving my interest was and is: what is the effect of disorder on phase transitions and on critical phenomena? So the result is that, if we look at these lectures from a technical viewpoint, they are about the specific class of statistical mechanics models that I call disordered pinning models. For this class we do have fairly satisfactory answers: essentially all the physical predictions on which there was a general consensus have now been established on firm mathematical grounds and there are now also rigorous result about some controversial physical statements. But, beyond the purely technical aspects, these notes are also an invitation to look beyond pinning models, that is, to more general statistical mechanics models.

It suffices to browse through these pages to realize that "more general statistical mechanics models" essentially reduces to the Ising model (and this is still, definitely, too much for these notes). The Ising model is going to accompany us along the various steps, but (hopefully!) in a way that it is not too invasive: the reader who is only interested in pinning phenomena should be able to follow leaving aside the sections on the Ising model, in which the presentation is rather informal. The choice of keeping disordered Ising models issues at an informal level is also due to the lack of rigorous results, in spite of some absolutely impressive achievements, and these portions of the notes present a number of open problems, which are most probably really challenging.

I would like to stress that since (polymer, interface, Markov process, etc.) pinning models are the means and not the aim, the modeling aspects and the very many variations or closely related classes of models are reduced to remarks or are even neglected entirely unless directly related to the main line of the notes. In this sense these notes do not review, for example, the vast literature on polymer models, not even that on general pinning or localization phenomena.

Moreover, these notes do not include hierarchical models on diamond lattices. Choices had to be made and this was the most painful one for at least two reasons: on one hand, part of the results and of the phenomena that I present have been obtained or have been understood first in the hierarchical set-up and, on the other, these somewhat exotic models definitely have a particular inner beauty.

I am presenting the combined work of several persons, to whom I am deeply grateful and indebted. I want to especially thank my closest collaborators and the people with whom I have discussed the subject of these notes most: Francesco Caravenna, Hubert Lacoin and Fabio Toninelli. Moreover special thanks are due to Bernard Derrida, who shaped my vision of the Harris criterion and who helped me in going through the physics literature (needless to say, I take full responsibility for what I have written on physics issues and I am absolutely aware that Bernard would have put things differently). I certainly cannot forget that my interest in disordered models and in localization phenomena dates back to my valued interaction, which persisted through the years, with Erwin Bolthausen.

Finally, I would like to give a big "thank you" to Lydia, Micol and Raika for their presence, in Saint-Flour, before and after.

Paris *Giambattista Giacomin*
March 2011

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Frequently Used Notations

$\mathbb{N}$	$\{1,2,3,\dots\}$
$a \wedge b$, $a \vee b$	$\min(a,b)$, $\max(a,b)$
$\lvert E\rvert$, $\mathscr{P}(E)$, $\mathbf{1}_E$	Cardinality, set of all subsets, indicator function of E
$\{a_n\}_n, \{b_n\}_n, \dots$	Sequences of real numbers
$a_n \sim b_n$	$\lim_n a_n/b_n = 1$
$a_n \approx b_n$	Used when one does not want, or cannot, be precise
$\tau = \{\tau_j\}_{j=0,1,\dots}$	Renewal sequence, often seen as subset of $\mathbb{N} \cup \{0\}$
$S = \{S_j\}_{j=0,1,\dots}$	Random walk
$\mathbf{P}$	Law of τ or law of S, according to the context
$K(n)$	$\mathbf{P}(\tau_1 = n)$, $n = 1,2,\dots,\infty$ (Chap. 2, Sect. 2.2)
$\overline{K}(n)$	$\sum_{j>n} K(n) \le 1$ ($n = 0,1,\dots$, sum does not include ∞)
$\widetilde{K}_h(n)$	(2.10) and Chap. 2, Sect. 2.2
$\widetilde{\tau}^{(h)}$	Renewal with inter-arrival law $\widetilde{K}_h(n)$
θ	Left shift operator: $(\theta a)_n = a_{n+1}$
h	Pinning potential
δ_n	$\mathbf{1}_{n\in\tau}$ (abuse of notation for $\mathbf{1}_\tau(n)$, here $\tau \subset \mathbb{N} \cup \{0\}$)
$Z_{N,h}$, $Z^{\mathtt{f}}_{N,h}$	Partition function of homogeneous system (constrained, free)
$\mathbf{P}_{N,h}$, $\mathbf{P}^{\mathtt{f}}_{N,h}$	Probability law of homogeneous system (constrained, free)
$\textsc{f}(h)$	Free energy of model with homogeneous pinning potential h
κ	correlation length
ω, $\mathbb{P}$	disorder or charge sequence, law of ω: Definition 3.1
$\mathrm{M}(\beta)$	$\mathbb{E}\exp(\beta\omega_1)$: Definition 3.1
β	disorder strength parameter
$Z_{N,\omega} = Z_{N,\omega,\beta,h}$	Partition function of disordered system (constrained)
$Z^{\mathtt{f}}_{N,\omega} = Z^{\mathtt{f}}_{N,\omega,\beta,h}$	Partition function of disordered system (free)
$\mathbf{P}_{N,\omega} = \mathbf{P}_{N,\omega,\beta,h}$	Disordered system probability (constrained)
$\mathbf{P}^{\mathtt{f}}_{N,\omega} = \mathbf{P}^{\mathtt{f}}_{N,\omega,\beta,h}$	Disordered system probability (free)
$\textsc{f}(\beta,h)$	Free energy of the disordered pinning model ($\textsc{f}(0,h) = \textsc{f}(h)$)
IID	Independent and Identically Distributed