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Classical Summation in Commutative and Noncommutative L_p-Spaces

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Preface

In the theory of orthogonal series the most important coefficient test for almost everywhere convergence of orthonormal series is the fundamental theorem of Menchoff and Rademacher. It states that whenever a sequence (α_k) of coefficients satisfies the "test" $\sum_k |\alpha_k \log k|^2 < \infty$, then for every orthonormal series $\sum_k \alpha_k x_k$ in $L_2(\mu)$ we have that $\sum_k \alpha_k x_k$ converges μ-almost everywhere. The aim of this research is to develop a systematic scheme which allows us to transform important parts of the now classical theory of almost everywhere summation of general orthonormal series in $L_2(\mu)$ into a similar theory for series in noncommutative L_p-spaces $L_p(\mathscr{M},\varphi)$ or even symmetric spaces $E(\mathscr{M},\varphi)$ constructed over a noncommutative measure space $(\mathscr{M},\varphi)$, a von Neumann algebra $\mathscr{M}$ of operators acting on a Hilbert space H together with a faithful normal state φ on this algebra.

In Chap. 2 we present a new and modern understanding of the classical theory on pointwise convergence of orthonormal series in the Hilbert spaces $L_2(\mu)$, and show that large parts of the classical theory transfer to a theory on pointwise convergence of unconditionally convergent series in spaces $L_p(\mu,X)$ of μ-integrable functions with values in Banach spaces X, or more generally Banach function spaces $E(\mu,X)$ of X-valued μ-integrable functions. Here our tools are strongly based on Grothendieck's metric theory of tensor products and in particular on his théorème fondamental. In Chap. 3 this force turns out to be even strong enough to extend our scheme to the setting of symmetric spaces $E(\mathscr{M},\varphi)$ of operators and Haagerup L_p-spaces $L_p(\mathscr{M},\varphi)$. In comparison with the old classical commutative setting the new noncommutation setting highlights new phenomena, and our theory as a whole unifies, completes and extends various results, both in the commutative and in the noncommutative world.

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