Lecture Notes in Mathematics 2019

Subseries:
École d'Été de Probabilités de Saint-Flour

Saint-Flour Probability Summer School

The Saint-Flour volumes are reflections of the courses given at the Saint-Flour Probability Summer School. Founded in 1971, this school is organised every year by the Laboratoire de Mathématiques (CNRS and Université Blaise Pascal, Clermont-Ferrand, France). It is intended for PhD students, teachers and researchers who are interested in probability theory, statistics, and in their applications.

The duration of each school is 13 days (it was 17 days up to 2005), and up to 70 participants can attend it. The aim is to provide, in three high-level courses, a comprehensive study of some fields in probability theory or Statistics. The lecturers are chosen by an international scientific board. The participants themselves also have the opportunity to give short lectures about their research work.

Participants are lodged and work in the same building, a former seminary built in the 18th century in the city of Saint-Flour, at an altitude of 900 m. The pleasant surroundings facilitate scientific discussion and exchange.

The Saint-Flour Probability Summer School is supported by:

- Université Blaise Pascal
- Centre National de la Recherche Scientifique (C.N.R.S.)
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For more information, see back pages of the book and
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Topological Complexity of Smooth Random Functions

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Preface

Before you start reading them, we should tell you something about what you can expect to find in these lecture notes, and what you should not be looking for.

First and foremost, you should keep in mind that what we have here was written to be a companion for the Saint Flour Lectures, which cover twelve hours of lecture time in eight meetings. This is sufficient time to be able to give a good introduction to a subject, but it is not enough time to either teach it in depth or describe all its ramifications and applications. The notes reflect both the challenge and limitation inherent in these parameters.

The second point to keep in mind is that the title of the notes and the lectures is overly optimistic. When originally planning the lectures we had ambitious plans regarding the material that we hoped to cover. Eventually we managed to internalise the fact that there was only so much one could do in twelve hours and so the “smooth random functions” of the title are limited to Gaussian, and Gaussian-related (to be defined later), random functions. As you will see, “topological complexity” is also somewhat of a grandiose over-statement. However, perhaps by the time you finish reading the notes, especially Chap. 6, you will at least have a feeling for what we originally had in mind.

Also related to the structure of these notes is the fact that in 2007 we published a 450 page Springer monograph [8] *Random Fields and Geometry* (hereafter *RFG*) which more or less covers the theoretical aspects of these notes. Furthermore, we currently have on hand half of a second monograph [9] that, when completed, will cover a wide range of applications. This one will be called *Applications of Random Fields and Geometry: Foundations and Case Studies* (hereafter *ARFG*). Despite the tragic loss of our *ARFG* co-author Keith Worsley in February 2009, we plan still to complete this book, which will also be published by Springer. At the moment the earlier chapters, which include homework exercises for some of the material of these notes, can be found on our web pages.

Since this second volume will also probably grow to a size comparable to its theoretical precursor, one wonders if we really can have anything new or different to say in the 100 pages or so of these notes.

In some cases, we do. There have been developments of the theory since the first monograph came out, and some of these are touched on here. In particular, Sect. 4.10 describes a powerful, infinite dimensional, extension of the Gaussian kinematic formula worked out in detail in [82]. Chapter 5 will point you in the directions of applications, and will, hopefully, one day form the core of *ARFG*. Finally in Chap. 6 there is a brief discussion of some brand new results at the interface of random fields and *algebraic* topology. It is these topics that originally motivated the title of these notes, but, somehow, at Saint Flour there was not enough time to discuss them in any detail.

However, the main advantage of these notes is precisely that they are neither as exhaustive nor, we hope, as exhausting, as the two monographs.

Our main aim here then will be to give a readable and easily accessible introduction to an area that has been developing rapidly over the past few years, without getting bogged down with too much technical detail. For the missing details in the theory you can turn, for the most part, to *RFG*, and, for more details on applications, to *ARFG*.

Our secondary aim is to motivate you to download the existing chapters of *ARFG* and help us debug them, and, of course, to motivate you to order a copy of *RFG* from Springer.[1]

Finally, we have some acknowledgments to make. The first is to the Saint Flour scientific committee, for originally inviting RJA to give the course. RJA delivered most of the lectures, but JET also carried some of the load. Adding to this the fact that these notes are heavily based on our joint monograph *RFG*, the result is the current joint, yet again, product.

We are also grateful to the agencies that supported our research and writing during the last 2–3 years. In particular, JET thanks the National Science Foundation (DMS-0906801), RJA thanks the Israel Science Foundation (853/10) and both thank the Binational Science Foundation (2008262) and the NSF for a SGER grant (also with Shmuel Weinberger) that had a lot to do with getting Chap. 6 written.

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[1] By the way, the grandfather of both of these books was published in 1981 as *The Geometry of Random Fields* [2], and after being out of print for many years is newly available in the SIAM series *Classics in Applied Mathematics*. It is, of course, rather dated, but also rather readable, because in those days its author had not yet learnt how to make easy material look hard. Since then, he has.

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