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Lebesgue and Sobolev Spaces with Variable Exponents

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Preface

In the past few years the subject of variable exponent spaces has undergone a vast development. Nevertheless, the standard reference is still the article by Kováčik and Rákosník from 1991. This paper covers only basic properties, such as reflexivity, separability, duality and first results concerning embeddings and density of smooth functions. In particular, the boundedness of the maximal operator, proved by Diening in 2002, and its consequences are missing.

Naturally, progress on more advanced properties is scattered in a large number of articles. The need to introduce students and colleagues to the main results led around 2005 to some short survey articles. Moreover, Diening gave lectures at the University of Freiburg in 2005 and Růžička gave a course in 2006 at the Spring School NAFSA 8 in Prague. The usefulness of a more comprehensive treatment was clear, and so we decided in the summer of 2006 to write a book containing both basic and advanced properties, with improved assumptions. Two further lecture courses were given by Hästö based on our material in progress (2008 in Oulu and 2009 at the Spring School in Paseky); another summary is Diening's 2007 habilitation thesis.

It has been our goal to make the book accessible to graduate students as well as a valuable resource for researchers. We present the basic and advanced theory of function spaces with variable exponents and applications to partial differential equations. Not only do we summarize much of the existing literature but we also present new results of our most recent research, including unifying approaches generated while writing the book.

Writing such a book would not have been possible without various sources of support. We thank our universities for their hospitality and the Academy of Finland and the DFG research unit "Nonlinear Partial Differential Equations: Theoretical and Numerical Analysis" for financial support. We also wish to express our appreciation of our fellow researchers whose results are presented and ask for understanding for the lapses, omissions and misattributions that may have entered the text. Thanks are also in order to Springer Verlag for their cooperation and assistance in publishing the book.

We thank our friends, colleagues and especially our families for their continuous support and patience during the preparation of this book.

Finally, we hope that you find this book useful in your journey into the world of variable exponent Lebesgue and Sobolev spaces.

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