

# Lecture Notes in Mathematics

2010

## **Editors:**

J.-M. Morel, Cachan

F. Takens, Groningen

B. Teissier, Paris



**FONDAZIONE  
CIME**  
ROBERTO CONTI  
CENTRO INTERNAZIONALE MATEMATICO ESTIVO  
INTERNATIONAL MATHEMATICAL SUMMER CENTER

Fondazione C.I.M.E., Firenze

C.I.M.E. stands for *Centro Internazionale Matematico Estivo*, that is, International Mathematical Summer Centre. Conceived in the early fifties, it was born in 1954 in Florence, Italy, and welcomed by the world mathematical community: it continues successfully, year for year, to this day.

Many mathematicians from all over the world have been involved in a way or another in C.I.M.E.'s activities over the years. The main purpose and mode of functioning of the Centre may be summarised as follows: every year, during the summer, sessions on different themes from pure and applied mathematics are offered by application to mathematicians from all countries. A Session is generally based on three or four main courses given by specialists of international renown, plus a certain number of seminars, and is held in an attractive rural location in Italy.

The aim of a C.I.M.E. session is to bring to the attention of younger researchers the origins, development, and perspectives of some very active branch of mathematical research. The topics of the courses are generally of international resonance. The full immersion atmosphere of the courses and the daily exchange among participants are thus an initiation to international collaboration in mathematical research.

C.I.M.E. Director  
Pietro ZECCA  
Dipartimento di Energetica "S. Stecco"  
Università di Firenze  
Via S. Marta, 3  
50139 Florence  
Italy  
e-mail: zecca@unifi.it

C.I.M.E. Secretary  
Elvira MASCOLO  
Dipartimento di Matematica "U. Dini"  
Università di Firenze  
viale G.B. Morgagni 67/A  
50134 Florence  
Italy  
e-mail: mascolo@math.unifi.it

For more information see CIME's homepage: <http://www.cime.unifi.it>

CIME activity is carried out with the collaboration and financial support of:  
– EMS - European Mathematical Society

Angiolo Farina · Axel Klar · Robert M.M. Mattheij  
Andro Mikelić · Norbert Siedow

# Mathematical Models in the Manufacturing of Glass

C.I.M.E. Summer School,  
Montecatini Terme, Italy 2008

Editor:  
Antonio Fasano



 Springer



*Editor*

Antonio Fasano  
Università degli Studi di Firenze,  
Dipartimento di Matematica “Ulisse Dini”  
Viale Morgagni 67/A  
I-50134 Firenze  
Italy  
fasano@math.unifi.it

*Authors: see List of Contributors*

ISBN: 978-3-642-15966-4 e-ISBN: 978-3-642-15967-1

DOI: 10.1007/978-3-642-15967-1

Springer Heidelberg Dordrecht London New York

Lecture Notes in Mathematics ISSN print edition: 0075-8434

ISSN electronic edition: 1617-9692

Mathematics Subject Classification (2010): 76D05, 76D07, 76B10, 76B45, 76M10, 80A20, 80A23, 35K05, 35K60, 35R35

© Springer-Verlag Berlin Heidelberg 2011

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable to prosecution under the German Copyright Law.

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

*Cover design:* SPi Publisher Services

Printed on acid-free paper

Springer is part of Springer Science+Business Media ([www.springer.com](http://www.springer.com))

# Preface

The EMS-CIME Course on Mathematical Models in the Manufacturing of Glass, Polymers and Textiles was held in Montecatini Terme (Italy) from September 8 to September 19, 2008. The course was co-directed by John Ockendon (OCIAM, Oxford, UK) and myself. The following topics were treated:

- (1) *Nonisothermal flows and fibres drawing* (Angiolo Farina and Antonio Fasano, Univ. Firenze, Italy, Andro Mikelic, Univ. Lyon, France) (\*)
- (2) *The mathematics of glass sheets and fibres* (Peter Howell, OCIAM, Oxford, UK)
- (3) *Radiative heat transfer in glass industry: modelling, simulation and optimisation* (Axel Klar and Norbert Siedow, ITWM – Fraunhofer, Kaiserslautern Germany) (\*)
- (4) *Modelling and simulation of glass forming processes* (Robert Mattheij, TU Eindhoven, The Netherlands) (\*)
- (5) *Injection moulding* (Hilary Ockendon, OCIAM, Oxford, UK)
- (6) *Fibre assembly modelling* (Hilary Ockendon, OCIAM, Oxford, UK)
- (7) *The mathematics of the windscreen sag process* (John Ockendon, OCIAM, Oxford, UK)

The focus was largely on glass manufacturing processes, with some digression to polymers and textile fibres in a context very close to the area of glass manufacturing. This volume collects the lecture notes of the courses marked with (\*), all devoted to problems in glass industry. It is regrettable that the other lecturers could not provide a chapter, because the subjects they illustrated were extremely interesting.

John Ockendon presented a fascinating and quite difficult problem: the production of a windscreen by the natural bending under gravity of a still soft glass layer clumped at the boundary. The audience was very excited by his colourful explanation of the underlying mechanics, making use of any deformable object he had at hand.

Hilary Ockendon posed stimulating questions about injection moulding and the “flow” of fibres in a fluffy tuft subject to traction. We had exciting afternoon sessions discussing such problems.

Peter Howell gave a series of lectures on the manufacturing of glass sheets and fibres which provided an excellent complementary view of the subjects treated by Farina, Fasano, Mikelic.

Indeed he addressed different problems in the same area (e.g.: how to get a fibre of a desired cross section), each with a different mathematical approach.

Fortunately most of the material not included in this volume is retrievable on the CIME web site, either in the form of slides or of excerpts from books.

Altogether the Course presented a remarkable review of quite advanced technological problems in the glass industry and of the mathematics involved. It was quite amazing to realize that such a seemingly small research area is on the contrary extremely rich and it calls for an impressively large variety of mathematical methods.

Despite the fact that the volume is not collecting all the material presented at the Course, it deals with a number of problems which are very typical in the field of glass manufacturing and it can certainly be useful not only to applied mathematicians, but also to physicists and engineers, who can find in it an overview of the most advanced models and methods.

The Chapter by J.A.W.M. Groot, R.M.M. Mattheij, and K.Y. Laevsky illustrates the various processes of glass forming, starting from the basic physical information, developing the mathematical models for each process, and analyzing the procedures of numerical computations.

Then we have two Chapters on radiative heat transfer in glass. The first one is by M. Frank and A. Klar, treating in detail the physics of radiation in glass and various approximated methods to model it, with an eye to numerical complexity. This is a quite substantial piece of work, due to the extension and the intrinsic difficulty of the problem. It is followed by the contribution of N. Siedow, who, after continuing the investigation of numerical methods for heat transfer problems including radiation and convection, passes to a question of great importance: the measurement of glass temperature from the observation of the spectrum of emitted optical radiation. From the mathematical point of view this is formulated as an inverse problem, which is typically ill posed.

The way of circumventing this difficulty is explained in detail and examples are provided.

The last Chapter, by A. Farina, A. Fasano, A. Mikelic, deals with the industrial process of glass fibre drawing, which goes through several stages having different thermal and mechanical characterizations, and analyzes in general non-isothermal motions of viscous fluids which are mechanically incompressible and thermally expandable.

I must abstain from commenting the scientific level of the present volume, since I am among the contributors, but at least I wish to express my deep gratitude to the Authors for their valuable work. Finally, also on behalf of John Ockendon, I wish to thank EMS and CIME for having made this Course possible. A particular thank to the Secretary of CIME, Prof. Elvira Mascolo, who took care of so many details.

*Antonio Fasano*

# Contents

<b>Mathematical Modelling of Glass Forming Processes</b> .....	1
J.A.W.M. Groot, Robert M.M. Mattheij, and K.Y. Laevsky	
1 Introduction .....	2
1.1 Glass Forming .....	2
1.2 Process Simulation .....	6
1.3 Outline .....	7
2 Mathematical Model .....	8
2.1 Geometry, Problem Domains and Boundaries .....	8
2.2 Balance Laws .....	9
3 Parison Press Model .....	16
3.1 Mathematical Model .....	16
3.2 Slender-Geometry Approximation .....	18
3.3 Motion of the Plunger .....	23
3.4 Simulation Model .....	26
3.5 Results .....	30
4 Blow Model .....	31
4.1 Mathematical Model .....	32
4.2 Glass-Air Interfaces .....	36
4.3 Variational Formulation .....	39
4.4 Simulation Model .....	42
4.5 Results .....	45
5 Direct Press Model .....	46
5.1 Mathematical Model .....	47
5.2 Simulation Model .....	49
5.3 Results .....	51
References .....	53
 <b>Radiative Heat Transfer and Applications for Glass Production Processes</b> .....	57
Martin Frank and Axel Klar	
1 Introduction .....	57
2 Radiative Heat Transfer Equations for Glass .....	58
2.1 Fundamental Quantities .....	59

2.2	Blackbody Radiation .....	61
2.3	The Transfer Equation.....	62
2.4	Overall Energy Conservation .....	65
2.5	Boundary Conditions.....	66
2.6	Summary .....	68
3	Direct Numerical Methods .....	70
3.1	Ordinates and Space Discretizations .....	71
3.2	Linear System Formulation .....	73
3.3	Preconditioning Techniques.....	77
3.4	A Fast Multilevel Preconditioner .....	82
3.5	Numerical Results .....	85
4	Higher-Order Diffusion Approximations.....	92
4.1	Asymptotic Analysis and Derivation of the $SP_N$ Approximations .....	94
4.2	Boundary Conditions for $SP_N$ Approximations.....	100
5	Moment Models .....	104
5.1	Spherical Harmonics .....	105
5.2	Minimum Entropy Closure.....	107
5.3	Flux-Limited Diffusion and Entropy Minimization .....	108
5.4	Partial Moments .....	110
5.5	Partial Moment $P_N$ Closure .....	112
5.6	Partial Moment Entropy Closure .....	113
6	Frequency-Averaged Moment Equations.....	115
6.1	Entropy Minimization .....	116
6.2	Inversion of the System .....	117
6.3	Properties .....	118
7	Numerical Comparisons.....	119
7.1	Numerical Results .....	119
7.2	Grey Transport.....	119
7.3	Grey Cooling .....	120
7.4	Multigroup Transport.....	123
7.5	Multigroup Cooling .....	125
7.6	Adaptive methods for the Simulation of 2-d and 3-d Cooling Processes.....	125
	References .....	131

## **Radiative Heat Transfer and Applications for Glass Production**

### **Processes II .....** 135

Norbert Siedow

1	Introduction .....	135
2	Models for Fast Radiative Heat Transfer Simulation .....	137
2.1	Introduction .....	137
3	Indirect Temperature Measurement of Hot Glasses.....	148
3.1	Introduction .....	148



3.2	The Basic Equation of Spectral Remote Temperature Sensing .....	149
3.3	Some Basics of Inverse Problems .....	150
3.4	Spectral Remote Sensing .....	159
3.5	Reconstruction of Initial Temperature .....	161
3.6	Conclusions .....	170
	References .....	170

## **Non-Isothermal Flow of Molten Glass: Mathematical**

### **Challenges and Industrial Questions .....**

Angiolo Farina, Antonio Fasano, and Andro Mikelić

1	Introduction .....	173
2	Mathematical Modelling .....	176
2.1	Definitions and Basic Equations .....	176
2.2	Fluids Physical Properties and Constitutive Equations .....	177
2.3	The General Model .....	181
2.4	Scaling and Dimensionless Formulation .....	183
3	Study of the Stationary Non-Isothermal Molten Glass Flow in a Die ....	187
3.1	Existence and Uniqueness Result for the Stationary Problem...	189
3.2	Oberbeck–Boussinesq Model .....	194
4	Modelling the Viscous Jet at the Exit of the Die .....	198
4.1	Definition of $L$ and Jet's Profile at the End of Stage ( $c$ ) .....	201
5	Terminal Phase of the Fiber Drawing .....	207
5.1	Derivation of the Model of Matovich–Pearson for the Thermal Case .....	209
5.2	Solvability of the Boundary Value Problems for the Stationary Effective Equations .....	216
	References .....	223

### **List of Participants .....**

225



# Contributors

**Angiolo Farina** Università degli Studi di Firenze, Dipartimento di Matematica “Ulisse Dini”, Viale Morgagni 67/A, I-50134 Firenze, Italy [farina@math.unifi.it](mailto:farina@math.unifi.it)

**Antonio Fasano** Università degli Studi di Firenze, Dipartimento di Matematica “Ulisse Dini”, Viale Morgagni 67/A, I-50134 Firenze, Italy [fasano@math.unifi.it](mailto:fasano@math.unifi.it)

**Martin Frank** University of Kaiserslautern, Erwin-Schrödinger-Strasse, 67663 Kaiserslautern, Germany [frank@mathematik.uni-kl.de](mailto:frank@mathematik.uni-kl.de)

**J.A.W.M. Groot** Department of Mathematics and Computer Science, Eindhoven University of Technology, PO Box 513, 5600 MB Eindhoven, The Netherlands [j.a.w.m.groot@tue.nl](mailto:j.a.w.m.groot@tue.nl)

**Axel Klar** University of Kaiserslautern, Erwin-Schrödinger-Strasse, 67663 Kaiserslautern, Germany and Fraunhofer ITWM, Fraunhofer Platz 1, 67663 Kaiserslautern, Germany [klar@itwm.fhg.de](mailto:klar@itwm.fhg.de)

**K.Y. Laevsky** [konstantin.laevsky@asml.com](mailto:konstantin.laevsky@asml.com)

**Robert M.M. Mattheij** Department of Mathematics and Computer Science, Eindhoven University of Technology, PO Box 513, 5600 MB Eindhoven, The Netherlands [r.m.m.mattheij@tue.nl](mailto:r.m.m.mattheij@tue.nl)

**Andro Mikelić** Université de Lyon, Lyon, F-69003, FRANCE; Université Lyon 1, Institut Camille Jordan, UMR 5208 CNRS, Bât. Braconnier, 43, Bd du onze novembre 1918 69622 Villeurbanne Cedex, FRANCE  
[Andro.Mikelic@univ-lyon1.fr](mailto:Andro.Mikelic@univ-lyon1.fr)

**Norbert Siedow** Fraunhofer-Institut für Techno- und Wirtschaftsmathematik Kaiserslautern, Germany [norbert.siedow@itwm.fraunhofer.de](mailto:norbert.siedow@itwm.fraunhofer.de)

