Lecture Notes in Mathematics 2010

Fondazione C.I.M.E., Firenze

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Preface

The EMS-CIME Course on Mathematical Models in the Manufacturing of Glass, Polymers and Textiles was held in Montecatini Terme (Italy) from September 8 to September 19, 2008. The course was co-directed by John Ockendon (OCIAM, Oxford, UK) and myself. The following topics were treated:

(1) *Nonisothermal flows and fibres drawing* (Angiolo Farina and Antonio Fasano, Univ. Firenze, Italy, Andro Mikelic, Univ. Lyon, France) (*)
(2) *The mathematics of glass sheets and fibres* (Peter Howell, OCIAM, Oxford, UK)
(3) *Radiative heat transfer in glass industry: modelling, simulation and optimisation* (Axel Klar and Norbert Siedow, ITWM – Fraunhofer, Kaiserslautern Germany) (*)
(4) *Modelling and simulation of glass forming processes* (Robert Mattheij, TU Eindhoven, The Netherlands) (*)
(5) *Injection moulding* (Hilary Ockendon, OCIAM, Oxford, UK)
(6) *Fibre assembly modelling* (Hilary Ockendon, OCIAM, Oxford, UK)
(7) *The mathematics of the windscreen sag process* (John Ockendon, OCIAM, Oxford, UK)

The focus was largely on glass manufacturing processes, with some digression to polymers and textile fibres in a context very close to the area of glass manufacturing. This volume collects the lecture notes of the courses marked with (*), all devoted to problems in glass industry. It is regrettable that the other lecturers could not provide a chapter, because the subjects they illustrated were extremely interesting.

John Ockendon presented a fascinating and quite difficult problem: the production of a windscreen by the natural bending under gravity of a still soft glass layer clumped at the boundary. The audience was very excited by his colourful explanation of the underlying mechanics, making use of any deformable object he had at hand.

Hilary Ockendon posed stimulating questions about injection moulding and the "flow" of fibres in a fluffy tuft subject to traction. We had exciting afternoon sessions discussing such problems.

Peter Howell gave a series of lectures on the manufacturing of glass sheets and fibres which provided an excellent complementary view of the subjects treated by Farina, Fasano, Mikelic.

Indeed he addressed different problems in the same area (e.g.: how to get a fibre of a desired cross section), each with a different mathematical approach.

Fortunately most of the material not included in this volume is retrievable on the CIME web site, either in the form of slides or of excerpts from books.

Altogether the Course presented a remarkable review of quite advanced technological problems in the glass industry and of the mathematics involved. It was quite amazing to realize that such a seemingly small research area is on the contrary extremely rich and it calls for an impressively large variety of mathematical methods.

Despite the fact that the volume is not collecting all the material presented at the Course, it deals with a number of problems which are very typical in the field of glass manufacturing and it can certainly be useful not only to applied mathematicians, but also to physicists and engineers, who can find in it an overview of the most advanced models and methods.

The Chapter by J.A.W.M. Groot, R.M.M. Mattheij, and K.Y. Laevsky illustrates the various processes of glass forming, starting from the basic physical information, developing the mathematical models for each process, and analyzing the procedures of numerical computations.

Then we have two Chapters on radiative heat transfer in glass. The first one is by M. Frank and A. Klar, treating in detail the physics of radiation in glass and various approximated methods to model it, with an eye to numerical complexity. This is a quite substantial piece of work, due to the extension and the intrinsic difficulty of the problem. It is followed by the contribution of N. Siedow, who, after continuing the investigation of numerical methods for heat transfer problems including radiation and convection, passes to a question of great importance: the measurement of glass temperature from the observation of the spectrum of emitted optical radiation. From the mathematical point of view this is formulated as an inverse problem, which is typically ill posed.

The way of circumventing this difficulty is explained in detail and examples are provided.

The last Chapter, by A. Farina, A. Fasano, A. Mikelic, deals with the industrial process of glass fibre drawing, which goes through several stages having different thermal and mechanical characterizations, and analyzes in general non-isothermal motions of viscous fluids which are mechanically incompressible and thermally expansible.

I must abstain from commenting the scientific level of the present volume, since I am among the contributors, but at least I wish to express my deep gratitude to the Authors for their valuable work. Finally, also on behalf of John Ockendon, I wish to thank EMS and CIME for having made this Course possible. A particular thank to the Secretary of CIME, Prof. Elvira Mascolo, who took care of so many details.

Antonio Fasano

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