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Preface

This is the fourth volume of the Paris-Princeton Lectures in Mathematical Finance. The goal of this series is to publish cutting edge research in self-contained articles prepared by established academics or promising young researchers invited by the editors. Contributions are refereed and particular attention is paid to the quality of the exposition, the goal being to publish articles that can serve as introductory references for research.

The series is a result of frequent exchanges between researchers in finance and financial mathematics in Paris and Princeton. Many of us felt that the field would benefit from timely exposés of topics in which there is important progress. René Carmona, Erhan Cinlar, Ivar Ekeland, Elyes Jouini, José Scheinkman and Nizar Touzi serve in the first editorial board of the Paris-Princeton Lectures in Financial Mathematics. Although many of the chapters involve lectures given in Paris or Princeton, we also invite other contributions. Springer Verlag kindly offered to host the initiative under the umbrella of the Lecture Notes in Mathematics series, and we are thankful to Catriona Byrne for her encouragement and her help.

This fourth volume contains five chapters. In the first chapter, Areski Cousin, Monique Jeanblanc, and Jean-Paul Laurent discuss risk management and hedging of credit derivatives. The latter are over-the-counter (OTC) financial instruments designed to transfer credit risk associated to a reference entity from one counterparty to another. The agreement involves a seller and a buyer of protection, the seller being committed to cover the losses induced by the default. The popularity of these instruments lead a runaway market of complex derivatives whose risk management did not develop as fast. This first chapter fills the gap by providing rigorous tools for quantifying and hedging counterparty risk in some of these markets.

In the second chapter, Stéphane Crépey reviews the general theory of forward backward stochastic differential equations and their associated systems of partial integro-differential obstacle problems and applies it to pricing and hedging financial derivatives. Motivated by the optimal stopping and optimal stopping game formulations of American option and convertible bond pricing, he discusses the well-posedness and sensitivities of reflected and doubly reflected Markovian Backward Stochastic Differential Equations. The third part of the paper is devoted to the variational inequality formulation of these problems and to a detailed discussion of viscosity solutions. Finally he also considers discrete path-dependence issues such as dividend payments.

The third chapter written by Olivier Guéant Jean-Michel Lasry and Pierre-Louis Lions presents an original and unified account of the theory and the applications of the mean field games as introduced and developed by Lasry and Lions in a series of lectures and scattered papers. This chapter provides systematic studies illustrating the application of the theory to domains as diverse as population behavior (the so-called Mexican wave), or economics (management of exhaustible resources). Some of the applications concern optimization of individual behavior when interacting with a large population of individuals with similar and possibly competing objectives. The analysis is also shown to apply to growth models and for example, to their application to salary distributions.

The fourth chapter is contributed by David Hobson. It is concerned with the applications of the famous Skorohod embedding theorem to the proofs of model independent bounds on the prices of options. Beyond the obvious importance of the financial application, the value of this chapter lies in the insightful and extremely pedagogical presentation of the Skorohod embedding problem and its application to the analysis of martingales with given one-dimensional marginals, providing a one-to-one correspondence between candidate price processes which are consistent with observed call option prices and solutions of the Skorokhod embedding problem, extremal solutions leading to robust model independent prices and hedges for exotic options.

The final chapter is concerned with pricing and hedging in exponential Lévy models. Peter Tankov discusses three aspects of exponential Lévy models: absence of arbitrage, including more recent results on the absence of arbitrage in multi-dimensional models, properties of implied volatility, and modern approaches to hedging in these models. It is a self contained introduction surveying all the results and techniques that need to be known to be able to handle exponential Lévy models in finance.

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The Editors

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