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The Analysis of Fractional Differential Equations

An Application-Oriented Exposition Using Differential Operators of Caputo Type

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Preface

There is a universe of mathematics lying in between the complete differentiations and integrations.

— O. Heaviside

This book is devoted to some questions in Fractional Calculus, that is, the theory of differential and integral operators of non-integer order, and in particular to differential equations containing such operators. Even though the first steps of the theory itself date back to the first half of the nineteenth century, the subject only really came to life over the last few decades. A particular feature is that engineers and scientists have developed new models that involve fractional differential equations. These models have been applied successfully, e.g., in mechanics (theory of viscoelasticity and viscoplasticity), (bio-)chemistry (modelling of polymers and proteins), electrical engineering (transmission of ultrasound waves), medicine (modelling of human tissue under mechanical loads), etc. The mathematical theory seems to be lagging behind the needs of those applications but the wealth of applications indeed indicates the truth of the above quote from Heaviside [93, §437]. There are some books dealing with the aspects that can be summarized as the "pure mathematical" side of the problems without taking into consideration those questions that arise in the applications mentioned above, and some that the engineer's point of view without a rigorous mathematical justification of the ideas. This book attempts to fill the gap between these two approaches: We try to establish a mathematically sound theory of the differential equations that have been shown to be relevant in practice and provide a thorough mathematical analysis. In order to be self-contained, we repeat the fundamentals of fractional calculus before coming to the main topic. A particular goal of this book is to provide a solid foundation that may later be used for the construction of efficient and reliable numerical methods for fractional differential equations. The author strongly believes that a successful development and a thorough understanding of such numerical schemes is not possible without such a stable analytical background.

The reader is assumed to be familiar with classical calculus (differential and integral calculus and the elementary theory of differential equations). A working knowledge of Lebesgue integration theory is helpful now and then, but not absolutely essential.

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Parts of the book have been used as a text for a graduate course on fractional differential equations that I taught to students of mathematics, physics and engineering at Technische Universität Braunschweig.

Braunschweig, June 2010 *Kai Diethelm*

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