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Controllability of Partial Differential Equations Governed by Multiplicative Controls

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To Irina, Elena and Dasha

Foreword

In a typical mathematical model of a controlled distributed parameter process one usually finds either boundary or internal locally distributed controls to serve as the means to describe the effect of external actuators on the process at hand. However, these classical controls, entering the model equations as *additive* terms, are not suitable to deal with a vast array of processes that can change their principal intrinsic properties due to the control actions. Important examples here include (but not limited to) the *chain reaction*-type processes in biomedical, nuclear, chemical and financial applications, which can change their (reaction) rate when certain "catalysts" are applied, and the so-called "smart materials", which can, for instance, alter their frequency response.

The goal of this monograph is to address the issue of global controllability of partial differential equations in the context of *multiplicative (or bilinear) controls*, which enter the model equations as coefficients. The mathematical models of our interest include the linear and nonlinear parabolic and hyperbolic PDE's, the Schrödinger equation, and coupled hybrid nonlinear distributed parameter systems associated with the swimming phenomenon.

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Alexander Khapalov

Preface

This monograph developed from the research conducted in 2001–2009 in the area of controllability theory of partial differential equations. The concept of controllability is a principal component of Control Theory which was brought to life in the 1950's by numerous applications in engineering, and has received the most significant attention both from the engineering and the mathematical communities since then.

A typical control problem deals with an evolution process which can be affected by a certain parameter, called control. Normally, the goal of a control problem is to steer this process from the given initial state to the desirable target state by selecting a suitable control among available options. If this is indeed possible, then one usually desires to achieve this steering while optimizing a certain criterion, solving what is called an optimal control problem.

Controllability theory studies the first part of the above-described control process. Namely, given any initial state, it studies the richness of the range of the mapping: *control* $\rightarrow$ *state of the process* (at some moment of time).

Controllability theory was originally developed in the 1960's for the linear ordinary differential equations, governed by the additive controls. Later, since the 1970's it became the subject of keen interest for the researchers working in the area of partial differential equations as well. As a result, nowadays there exists a quite comprehensive controllability theory for the linear pde's governed by the additive controls which can act inside the system's space domain (locally distributed or point controls) or on its boundary (boundary controls). In such context, the respective mathematical methods are essentially the methods of the theory of linear operators, particularly, of the duality theory.

In this monograph, however, the subjects of interest are the multiplicative controls that enter the system equations as coefficients. Therefore, the aforementioned *control* $\rightarrow$ *state of the process* mapping becomes highly nonlinear, even if the original pde is linear. This gives rise to the necessity of developing a different methodology for this type of controllability problems.

In this monograph we address this issue in the context of linear and semilinear parabolic and hyperbolic equations, as well as the Schrödinger equation. Particular attention is given to nonlinear swimming models. In the introduction we discuss the

motivation for the use of multiplicative controls as opposed to the classical additive ones, and compare the mathematical methods involved in the respective studies.

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