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Alberto Parmeggiani

Spectral Theory of Non-Commutative Harmonic Oscillators: An Introduction

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To Serena & Lorenzo, Luisa & Pier Luigi

Preface

This book grew out of a series of lectures given at the Mathematics Department of Kyushu University in the Fall 2006, within the support of the 21st Century COE Program (2003–2007) “Development of Dynamical Mathematics with High Functionality” (Program Leader: prof. Mitsuhiro Nakao).

It was initially published as the Kyushu University COE Lecture Note number 8 (COE Lecture Note, 8. Kyushu University, The 21st Century COE Program “DMHF”, Fukuoka, 2008. vi+234 pp.), and in the present form is an extended version of it (in particular, I have added a section dedicated to the Maslov index).

The book is intended as a rapid (though not so straightforward) *pseudodifferential* introduction to the spectral theory of certain systems, mainly of the form $a_2 + a_0$ where the entries of a_2 are homogeneous polynomials of degree 2 in the (x,ξ)-variables, $(x,\xi) \in \mathbb{R}^n \times \mathbb{R}^n$, and a_0 is a constant matrix, the so-called *non-commutative harmonic oscillators*, with particular emphasis on a class of systems introduced by M. Wakayama and myself about ten years ago. The class of non-commutative harmonic oscillators is very rich, and many problems are still open, and worth of being pursued.

I wish to thank Masato Wakayama, dearest friend and collaborator, and my friends and colleagues Nicola Arcozzi, Sandro Coriasco, Sandro Graffi, Frédéric Hérau, Takashi Ichinose, Miyuki Kuze, Lidia Maniccia, Luca Migliorini, Cesare Parenti and Cosimo Senni for their invaluable help in giving these notes a (hopefully) decent shape.

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Alberto Parmeggiani

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