Lecture Notes in Mathematics 1983

Habib Ammari (Ed.)

# Mathematical Modeling in Biomedical Imaging I

## Electrical and Ultrasound Tomographies, Anomaly Detection, and Brain Imaging

Springer

*Editor*
Habib Ammari
Laboratoire Ondes et Acoustique
ESPCI, 10 Rue Vauquelin
75231 Paris, Cedex 05
France
habib.ammari@espci.fr

and

Centre de Mathématiques Appliquées
Ecole Polytechnique
91128, Palaiseau, Cedex
France
habib.ammari@polytechnique.fr

ISBN: 978-3-642-03443-5 e-ISBN: 978-3-642-03444-2
DOI: 10.1007/978-3-642-03444-2
Springer Heidelberg Dordrecht London NewYork

Lecture Notes in Mathematics ISSN print edition: 0075-8434
ISSN electronic edition: 1617-9692

Library of Congress Control Number: 2009932673

Mathematics Subject Classification (2000): 92C55, 31A25, 31A10, 35B25, 35J55, 35J65, 34A55



*Cover design*: SPi Publisher Services

Printed on acid-free paper

springer.com

# Preface

Mathematical sciences are contributing more and more to advances in life science research, a trend that will grow in the future.

Realizing that the mathematical sciences can be critical to many areas of biomedical imaging, we organized a three-day minicourse on mathematical modelling in biomedical imaging at the Institute Henri Poincaré in Paris in March 2007. Prominent mathematicians and biomedical researchers were paired to review the state-of-the-art in the subject area and to share mathematical insights regarding future research directions in this growing discipline.

The speakers gave presentations on hot topics including electromagnetic brain activity, time-reversal techniques, elasticity imaging, infrared thermal tomography, acoustic radiation force imaging, electrical impedance and magnetic resonance electrical impedance tomographies. Indeed, they contributed to this volume with original chapters to give a wider audience the benefit of their talks and their thoughts on the field.

This volume is devoted to providing an exposition of the promising analytical and numerical techniques for solving important biomedical imaging problems and to piquing interest in some of the most challenging issues. We hope that it will stimulate much needed progress in the directions that were described during the course. The biomedical imaging problems addressed in this volume trigger the investigation of interesting and difficult problems in various branches of mathematics including partial differential equations, harmonic analysis, complex analysis, numerical analysis, optimization, image analysis, and signal theory.

The partial support offered by the ANR project EchoScan (AN-06-Blan-0089) is acknowledged. We also thank the staff at the Institute Henri Poincaré.

Paris
March 2009

*Habib Ammari*

# List of Contributors

**Habib Ammari** Laboratoire Ondes et Acoustique, 10 rue Vauquelin, CNRS, E.S.P.C.I, Université Paris 7, 75005, Paris, France, habib.ammari@espci.fr

**Sylvain Baillet** Cognitive Neuroscience and Brain Imaging Laboratory, Université Pierre and Marie Curie-Paris 6, CNRS UPR 640 – LENA, Paris, France, sylvain.baillet@chups.jussieu.fr

**George Dassios** Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 0WA, UK, G.Dassios@damtp.cam.ac.uk

**Mathias Fink** Laboratoire Ondes et Acoustique, 10 rue Vauquelin, CNRS, E.S.P.C.I, Université Paris 7, 75005, Paris, France, mathias.fink@espci.fr

**Hyeonbae Kang** Department of Mathematics, Inha University, Incheon 402-751, Korea, hbkang@inha.ac.kr

**Julien Lefèvre** Cognitive Neuroscience and Brain Imaging Laboratory, Université Pierre and Marie Curie-Paris 6, CNRS UPR 640 – LENA, Paris, France, julien.lefevre@chups.jussieu.fr

**Jin Keun Seo** Department of Mathematics, Yonsei University, 262 Seongsanno, Seodaemun, Seoul 120-749, Korea, seoj@yonsei.ac.kr

**Mickael Tanter** Laboratoire Ondes et Acoustique, 10 rue Vauquelin, CNRS, E.S.P.C.I, Université Paris 7, 75005, Paris, France, mickael.tanter@espci.fr

**Eung Je Woo** College of Electronics and Information, Kyung Hee University, Seoul 130-701, Korea, ejwoo@khu.ac.kr

# Contents

# Introduction

Medical imaging modalities such as computerized tomography using X-ray and magnetic resonance imaging have been well established providing three-dimensional high-resolution images of anatomical structures inside the human body. Computer-based mathematical methods have played an essential role for their image reconstructions. However, since each imaging modality has its own limitations, there have been much research efforts to expand our ability to see through the human body in different ways. Lately, biomedical imaging research has been dealing with new imaging techniques to provide knowledge of physiologic functions and pathological conditions in addition to structural information.

Electrical impedance tomography, ultrasound imaging, and electrical and magnetic source imaging are three of such attempts for functional imaging and monitoring of physiological events.

The aim of this book is to review the most recent advances in the mathematical and numerical modelling of these three emerging modalities. Although they use different physical principles for signal generation and detection, the underlying mathematics are quite similar. We put a specific emphasis on the mathematical concepts and tools for image reconstruction. Other promising modalities such as photo-acoustic imaging and fluorescence microscopy as well as those in nuclear medicine will be discussed in a forthcoming volume.

Electrical impedance tomography uses low-frequency electrical current to probe a body; the method is sensitive to changes in electrical conductivity. By injecting known amounts of current and measuring the resulting electrical potential field at points on the boundary of the body, it is possible to invert such data to determine the conductivity or resistivity of the region of the body probed by the currents. This method can also be used in principle to image changes in dielectric constant at higher frequencies. However, the aspect of the method that is most fully developed to date is the imaging of conductivity. Potential applications of electrical impedance tomography include determination of cardiac output, monitoring for pulmonary edema, and screening for breast cancer.

Electrical source imaging is an emerging technique for reconstructing brain electrical activity from electrical potentials measured away from the brain. The concept of electrical source imaging is to improve on electroencephalography by determining the locations of sources of current in the body from measurements of voltages.

Ion currents arising in the neurons of the brain produce magnetic fields outside the body that can be measured by arrays of superconducting quantum interference device detectors placed near the chest; the recording of these magnetic fields is known as magnetoencephalography. Magnetic source imaging is the reconstruction of the current sources in the brain from these recorded magnetic fields. These fields result from the synchronous activity of tens or hundreds of thousands of neurons.

Both magnetic source imaging and electrical source imaging seek to determine the location, orientation, and magnitude of current sources within the body.

Ultrasound imaging is a noninvasive, easily portable, and relatively inexpensive diagnostic modality which finds extensive use in the clinic. The major clinical applications of ultrasound include many aspects of obstetrics and gynecology involving the assessment of fetal health, intra-abdominal imaging of the liver, kidney, and the detection of compromised blood flow in veins and arteries.

Operating typically at frequencies between 1 and 10 MHz, ultrasound imaging produces images via the backscattering of mechanical energy from interfaces between tissues and small structures within tissue. It has high spatial resolution, particularly at high frequencies, and involves no ionizing radiation. The weakness of the technique include the relatively poor soft-tissue contrast and the fact that gas and bone impede the passage of ultrasound waves, meaning that certain organs can not easily be imaged.

As we said before, in this book not only the basic mathematical principles of these three emerging modalities are reviewed but also the most recent developments to improve them are reported. We emphasize the mathematical concepts and tools for image reconstruction. Our main focuses are, on one side, on promising anomaly detection techniques in electrical impedance tomography and in elastic imaging using the method of small-volume expansions and in ultrasound imaging using time-reversal techniques, and on the other side, on emerging multi-physics or hybrid imaging approaches such as the magnetic resonance electrical impedance, impediography, and magnetic resonance elastography.

The book is organized as follows. Chapter 1 is devoted to electrical impedance tomography and magnetic resonance electrical impedance tomography. It focuses on robust reconstructions of conductivity images under practical environments having various technical limitations of data collection equipments and fundamental limitations originating from its inherent ill-posed nature. The mathematical formulation of the magnetic resonance electrical impedance tomography and multi-frequencies electrical

impedance tomography are rigorously described. Efficient image reconstruction algorithms are provided and their limitations are discussed.

Chapter 2 outlines the basic physical principles of time-reversal techniques and their applications in ultrasound imaging. It gives a good introduction to this very interesting subject.

Chapter 3 covers the method of small-volume expansions. A remarkable feature of the method of small-volume expansions is that it allows a stable and accurate reconstruction of the location and of geometric features of the anomalies, even for moderately noisy data. Based on this method robust and efficient algorithms for imaging small thermal conductivity, electromagnetic, and elastic anomalies are provided. Emerging multi-physics or hybrid imaging approaches, namely impediography, magneto-acoustic imaging, and magnetic resonance elastography are also discussed. In these techniques, different physical types of radiation are combined into one tomographic process to alleviate deficiencies of each separate type of waves, while combining their strengths. Finally, a mathematical formulation of the concept of time reversing waves is provided and its use in imaging is described.

Chapter 4 deals with electrical and magnetic source imaging reconstruction methods for focal brain activity. Mathematical formulations and uniqueness and non-uniqueness results for the inversion source problems are given. The basic mathematical model is described by the Biot–Savart law of magnetism, which makes the mathematical difficulties for solving the inverse source problem very similar to those in magnetic resonance electrical impedance tomography discussed in Chap. 1.

Chapter 5 considers time-resolved imaging of brain activity. It discusses optical flow techniques in order to infer on the stability of brain activity.