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Potential Analysis of Stable Processes and its Extensions

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Foreword

This monograph is devoted to the potential theory of stable stochastic processes and related topics, such as the subordinate Brownian motions (including the relativistic process) and Feynman–Kac semigroups generated by certain Schrödinger operators.

The stable Lévy processes and related stochastic processes play an important role in stochastic modelling in applied sciences, in particular in financial mathematics, and the theoretical motivation for the study of their fine properties is also very strong. The potential theory of stable and related processes naturally extends the theory established in the classical case of the Brownian motion and the Laplace operator.

The foundations and general setting of probabilistic potential theory were given by G.A. Hunt [92](1957), R.M. Blumenthal and R.K. Gettoor [23](1968), S.C. Port and J.C. Stone [130](1971). K.L. Chung and Z. Zhao [62](1995) have studied the potential theory of the Brownian motion and related Schrödinger operators. The present book focuses on classes of processes that contain the Brownian motion as a special case. A part of this volume may also be viewed as a probabilistic counterpart of the book of N.S. Landkof [117](1972).

The main part of Introduction that opens the book is a general presentation of fundamental objects of the potential theory of the isotropic stable Lévy processes in comparison with those of the Brownian motion (presented in a subsection). The introduction is accessible to a non-specialist. Also the chapters that follow should be of interest to a wider audience. A detailed description of the content of the book is given at the end of Chapter 1.

Some of the material of the book was presented by T. Byczkowski, T. Kulczycki, M. Ryznar and Z. Vondraček at the Workshop on Stochastic and Harmonic Analysis of Processes with Jumps held at Angers, France, May 2-9, 2006. The authors are grateful to the organizers and to the main supporters of the Workshop – the CNRS, the European Network of Harmonic Analysis HARP and the University of Angers – for this opportunity, which gave the incentive to write the monograph.

The book was written while Z. Vondraček was visiting the Department of Mathematics of University of Illinois at Urbana-Champaign. He thanks the department for the stimulating environment and hospitality. Thanks are also due to Andreas Kyprianou for several useful comments. The editors thank T. Luks for critical reading of some parts of the manuscript and for some of the figures illustrating the text.

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