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J.-M. Morel, Cachan

F. Takens, Groningen

B. Teissier, Paris

Jan Christian Rohde

Cyclic Coverings, Calabi-Yau Manifolds and Complex Multiplication

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Jan Christian Rohde
Institut fuer Algebraische Geometrie
Leibniz Universität Hannover
Welfengarten 1, GRK 1463
30167 Hannover
Germany
rohde@math.uni-hannover.de

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Preface

Calabi-Yau manifolds have been an object of extensive research during the last two decades. One of the reasons is the importance of Calabi-Yau 3-manifolds in modern physics - notably string theory. An interesting class of Calabi-Yau manifolds is given by those with complex multiplication (CM). Calabi-Yau manifolds with CM are also of interest in theoretical physics, e.g. in connection with mirror symmetry and black hole attractors.

It is the main aim of this book to construct families of Calabi-Yau 3-manifolds with dense sets of fibers with complex multiplication. Most examples in this book are constructed using families of curves with dense sets of fibers with CM . The contents of this book can roughly be divided into two parts. The first six chapters deal with families of curves with dense sets of CM fibers and introduce the necessary theoretical background. This includes among other things several aspects of Hodge theory and Shimura varieties. Using the first part, families of Calabi-Yau 3-manifolds with dense sets of fibers with CM are constructed in the remaining five chapters. In the appendix one finds examples of Calabi-Yau 3-manifolds with complex multiplication which are not necessarily fibers of a family with a dense set of CM fibers.

The author hopes to have succeeded in writing a readable book that can also be used by non-specialists. On the other hand the expert will find new results about variations of Hodge structures and new examples of families of curves and Calabi-Yau manifolds with dense sets of fibers with CM . The author believes that this book will also be interesting for physicists.

This book is based on the authors doctoral thesis at Universität Duisburg-Essen. The author wishes to thank his former adviser Eckart Viehweg for his excellent guidance and support. The text has been revised for publication at the Graduiertenkolleg "Analysis, Geometrie und String Theorie" at Leibniz Universität Hannover.

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J. C. Rohde

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