

Polymers in Random Environment

OUTLINE:

In Part B we look at five models of polymers in random environment.

- In Chapter 8, we consider a polymer with positive and negative *charges*, arranged randomly along the chain, resulting in repulsion and attraction at the locations where the polymer intersects itself. This is a model for a polymer in an *electrolyte*: the charges on the polymer are surrounded by an ionic atmosphere that screens the charges so that they are felt “on site” only. Our main result will be that the *annealed* charged polymer is *subdiffusive* in any $d \geq 1$. We obtain the full scaling behavior. The annealed model is appropriate for polymers consisting of *amphoteric* monomers, i.e., monomers containing both acidic and basic functional groups, since such monomers may change their charge as the pH of the solution in which they are immersed is varied.
- In Chapter 9, we look at a copolymer, consisting of a random concatenation of *hydrophobic* and *hydrophilic* monomers, interacting with two solvents, *oil* and *water*, separated by a *linear* interface. If the interaction between monomers and solvents is strong and unbiased enough, then the polymer *localizes* near the interface, otherwise it *delocalizes* away from the interface. Our main result is a detailed analysis of the *quenched* critical curve separating the two phases, the order of the phase transition, and the path behavior in each of the two phases.
- In Chapter 10, we extend the results of Chapter 9 to a model where the linear interface is replaced by a *percolation-type* interface. This is a primitive model for a copolymer in an *emulsion* consisting of *mesoscopically* large droplets of oil floating in water. Various forms of (de)localization are possible, depending on whether the oil droplets percolate or not.
- In Chapter 11, the polymer interacts with a linear substrate composed of different types of atoms or molecules, occurring in a random order. Each time the polymer hits the substrate it picks up a reward or a penalty according to the type of atom or molecule it encounters. If the reward to

hit the substrate is strong enough, then the polymer *localizes*, otherwise it *delocalizes*. Both the *pinning* and the *wetting* version of the model are of interest. Our main result is a detailed analysis of the *quenched* critical curve separating the two phases, the order of the phase transition, and the path behavior in each of the two phases. Moreover, we investigate when the quenched and the annealed critical curve are different, referred to as *relevant disorder*, and when they coincide, referred to as *irrelevant disorder*. The wetting version of the model can be used to describe the *denaturation transition* of DNA, where the two strands detach with increasing temperature, as well as the *unzipping transition* of DNA under the action of a force.

- In Chapter 12, finally, we consider a polymer interacting with a *random potential* field. An example is a hydrophobic homopolymer living in a *micro-emulsion* consisting of *microscopically* small droplets of oil and water. Depending on the dimension and on the strength of the interaction, the polymer can be in a phase of *weak disorder* or a phase of *strong disorder*, where the polymer is diffusive, respectively, superdiffusive. Our main result is a classification of these two phases. Another application is a polymer in a *gel* with different sizes of pockets.

Except in Chapter 8, we will consider only *directed paths*, i.e., paths that cannot backtrack (already used in Section 6.2 and Chapter 7). This restriction is necessary in order to make the models mathematically tractable.

The presence of the random environment adds on a considerable layer of complication, and necessitates the development of new ideas and techniques. Some of these are successful in elucidating the behavior of the polymer, others only partially resolve key questions.

Polymers in random environment is a challenging and captivating area, with problems that are driven by specific applications. Many of these problems are “easy to state but hard to solve”, and frequently cannot be tackled at full power with non-rigorous methods (such as mean-field approximations and replica computations), in the best of the tradition of mathematical physics.

WARNING: In Part B we use the symbol ω to denote the random environment. Be careful to distinguish this from the symbol w that is used to denote the polymer path.