Part A

Polymers with Self-interaction

OUTLINE:

In Part A we look at a four variations on the two "plain vanilla" models described in Chapter 2.

- In Chapters 3 and 4, we consider a polymer for which self-intersections are not forbidden (as in SAW) but are penalized, resulting in a repulsive interaction modeling the effect of "steric hindrance". This model, which we call *soft polymer*, may be viewed as an "interpolation" between SRW and SAW. Our main result will be that the soft polymer is ballistic in $d = 1$ and diffusive in $d \geq 5$, like SAW. We will obtain the full scaling behavior.
- In Chapter 5, we consider a polymer for which self-intersections are penalized in a way that depends on their distance along the chain, in such a way that long loops are less penalized than short loops. This model, which we call *elastic polymer*, provides a crude way of incorporating the effect of "stiffness" of a polymer (i.e., its resistance against making sharp bends). We will show that the smaller the stiffness, the lower the critical dimension for diffusive behavior.
- In Chapter 6, we add a reward for self-touchings (pairs of monomers occupying neighboring sites), which results in a mix of repulsive and attractive interactions. This is a model for a polymer in a poor solvent. If the attraction is strong enough to compete with the repulsion, then the polymer *collapses* from a "random coil" to a "compact ball". We will show that there are three phases – extended, collapsed and localized – with different scaling properties.
- In Chapter 7, finally, we consider a polymer that interacts with a linear substrate: monomers at the substrate receive a reward. If the attraction is strong enough, then the polymer *adsorbs* onto the substrate. We consider both the case where the substrate is penetrable ("pinning by an interface") and where it is impenetrable ("wetting of a surface"). The latter is a model for paint on a wall: at low temperature the paint sticks to the wall, at high

temperature it does not. We also look at what happens when a *force* is applied to the right endpoint of an adsorbed polymer, pulling it away from and off the substrate with the help of optical tweezers. In addition, we look at a polymer in a slit between two impenetrable substrates and compute the effective force the polymer exerts on the substrates, which can be either attractive or repulsive. This is a model for colloidal dispersions of particles with polymers attached to them.

What is challenging about the models to be described below is that the polymers appear as random objects with a *long-range interaction*: monomers that are far apart along the chain can meet and can interact with each other. This places polymers in a league of their own, with specific questions driven by specific applications. As such, polymers are different from more classical probabilistic objects like Markov chains, percolation, the contact process or interacting diffusions. Consequently, their study requires the development of proper ideas and techniques.