Part V
Appendix: Technical Verifications

Foreword

The goal of this part is to achieve technical verifications of §12.3 and §15.2:

- we check condition (2) of theorem 12.1.4 to prove that the category of algebras over an operad $\mathtt{R}$ is equipped with a semi model structure,
- we prove that the bifunctor $(M, A) \mapsto \mathrm{S}_{\mathtt{R}}(M, A)$ associated to an operad $\mathtt{R}$ satisfies an analogue of the pushout-product axiom of tensor products in symmetric monoidal model categories.

As usual, we address the case of $\mathtt{R}$-algebras in any symmetric monoidal category $\mathcal{E}$ over the base category $\mathcal{C}$.

The bifunctor $(M, A) \mapsto \mathrm{S}_{\mathtt{R}}(M, A)$ does not preserves all colimits in A. Therefore we can not apply the argument of lemma 11.3.2 to prove the required pushout-product axiom.

To handle the difficulty we introduce right $\mathtt{R}$-modules $\mathrm{S}_{\mathtt{R}}[M, A]$ such that $\mathrm{S}_{\mathtt{R}}(M, A) = \mathrm{S}_{\mathtt{R}}[M, A](0)$ and we study pushout-products on the bifunctor $(M, A) \mapsto \mathrm{S}_{\mathtt{R}}[M, A]$. The same method is used in the proof that $\mathtt{R}$-algebras form a semi model category.