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Modules over Operads and Functors

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Preface

The notion of an operad was introduced 40 years ago in algebraic topology in order to model the structure of iterated loop spaces [6, 47, 60]. Since then, operads have been used fruitfully in many fields of mathematics and physics.

Indeed, the notion of an operad supplies both a conceptual and effective device to handle a variety of algebraic structures in various situations. Many usual categories of algebras (like the category of commutative and associative algebras, the category of associative algebras, the category of Lie algebras, the category of Poisson algebras, . . .) are associated to operads.

The main successful applications of operads in algebra occur in deformation theory: the theory of operads unifies the construction of deformation complexes, gives generalizations of powerful methods of rational homotopy, and brings to light deep connections between the cohomology of algebras, the structure of combinatorial polyhedra, the geometry of moduli spaces of surfaces, and conformal field theory. The new proofs of the existence of deformation-quantizations by Kontsevich and Tamarkin bring together all these developments and lead Kontsevich to the fascinating conjecture that the motivic Galois group operates on the space of deformation-quantizations (see [35]).

The purpose of this monograph is to study not operads themselves, but *modules over operads* as a device to model functors between categories of algebras as effectively as operads model categories of algebras.

Modules over operads occur naturally when one needs to represent universal complexes associated to algebras over operads (see [14, 54]).

Modules over operads have not been studied as extensively as operads yet. However, a generalization of the theory of Hopf algebras to modules over operads has already proved to be useful in various mathematical fields: to organize Hopf invariants in homotopy theory [2]; to study non-commutative generalizations of formal groups [12, 13]; to understand the structure of certain combinatorial Hopf algebras [38, 39]. Besides, the notion of a module over an operad unifies and generalizes classical structures, like Segal's notion of a

Γ-object, which occur in homological algebra and homotopy theory. In [33], Kapranov and Manin give an application of the relationship between modules over operads and functors for the construction of Morita equivalences between categories of algebras.

Our own motivation to write this monograph comes from homotopy theory: we prove, with a view to applications, that functors determined by modules over operads satisfy good homotopy invariance properties.

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