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Operator-Valued Measures and Integrals for Cone-Valued Functions

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Preface

The aim of this book is twofold: Firstly, to introduce the developing theory of locally convex cones to a wider audience. This theory generalizes locally convex topological vector spaces and permits many additional and substantially different examples and applications. In the aspects of the theory that have been developed so far, the increase in generality does not lead to any compromises with respect to the depth of its results. The main difference to vector spaces is the presence of infinity-type unbounded elements and the general non-availability of the cancellation law. Some important mathematical models, while close to the structure of vector spaces are of this type. They do not allow subtraction of their elements or multiplication by negative scalars. Examples are certain classes of set-valued or extended real-valued functions that may take infinite values. These arise naturally in integration theory, potential theory and in a variety of other settings and do not form vector spaces. Therefore many results and techniques from classical functional analysis can not be immediately applied. Locally convex cones carry a reflexive and transitive order relation, and their (convex semiuniform) topology is defined using this order structure. The first part of this book contains a review and summary of the aspects of the theory of locally convex cones that have been developed so far, sometimes without detailed proofs, but references to the sources instead. The theory is then developed further, adding some (hopefully) interesting new features.

This leads to the second objective: Locally convex cones are used to provide the setting for a novel approach to integration theory. The generality of their structure allows to deal simultaneously with a wide variety of situations, including extended real-valued, vector-valued, operator-valued and cone-valued measures and functions. Topological limits from the classical theory are replaced by approximations in terms of the order structure of a locally convex cone. The main results include convergence theorems for measures and functions and an integral representation theorem for continuous linear operators on certain cones of functions. The latter establishes that a given operator can be expressed as an integral with respect to some unique measure. This is a

very technical result and requires a lengthy proof. It is followed by a comprehensive collection of special cases and applications. Some of these lead to known representation results for compact and weakly compact operators on Banach space-valued functions, but the more general cases are new. The insertions of a special case yield the classical Spectral Representation Theorem for normal linear operators on a complex Hilbert space.

Contents