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A Minicourse on Stochastic Partial Differential Equations

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Preface

From May 8 to May 19th of 2006, the Department of Mathematics at the University of Utah hosted a minicourse on some modern topics in stochastic partial differential equations [SPDEs]. The participants included graduate students and recent PhDs from across North America, as well as research mathematicians at diverse stages of their careers. Intensive courses were given by Robert C. Dalang, Davar Khoshnevisan, An Le, Carl Mueller, David Nualart, Boris Rozovsky, and Yimin Xiao. The present book is comprised of most of those lectures.

For nearly three decades, the topic of SPDEs has been an area of active research in pure and applied mathematics, fluid mechanics, geophysics, and theoretical physics. The theory of SPDEs has a similar flavor as PDEs and interacting particle systems in the sense that most of the interesting developments generally evolve in two directions: There is the general theory; and then there are specific problem-areas that arise from concrete questions in applied science. As such, it is unlikely that there ever will be a cohesive all-encompassing theory of stochastic partial differential equations. With that in mind, the present volume follows the style of the Utah minicourse in SPDEs and attempts to present a selection of interesting themes within this interesting area. The presentation, as well as the choice of the topics, were motivated primarily by our desire to bring together a combination of methods and deep ideas from SPDEs (Chapters 1, 2, and 4) and Gaussian analysis (Chapters 3 and 5), as well as potential theory and geometric measure theory (Chapter 5). Ours is a quite novel viewpoint, and we believe that the interface of the mentioned theories is fertile ground that shows excellent potential for continued future research.

We are aware of at least four books on SPDEs that have appeared since we began to collect the material for this project [4; 8; 12; 14]. Although there is little overlap between those books and the present volume, the rapidly-growing number of books on different aspects of SPDEs represents continued, as well as a growing, interest in both the theory as well as the applications of the subject. The reader is encouraged to consult the references for examples

in: (i) Random media [2; 4; 18] and filtering theory [15]; (ii) applications in fluid dynamics and turbulence [1; 2; 17]; and (iii) in statistical physics of disordered media [2; 6; 7; 10]. Further references are scattered throughout the lectures that follow. The reader is invited to consult the references to this preface, together with their voluminous bibliographies, for some of the other viewpoints on this exciting topic.

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Salt Lake City, Utah
July 1, 2008

Davar Khoshnevisan
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References

[1] A. Bourlioux, A. J. Majda, and O. Volkov (2006). Conditional statistics for a passive scalar with a mean gradient and intermittency. *Phys. Fluids* **18***(10)*, 104102, 10 pp.

[2] René A. Carmona and S. A. Molchanov (1994). Parabolic Anderson problem and intermittency, *Mem. Amer. Math. Soc.* **108***(518)*

[3] *Stochastic Partial Differential Equations: Six Perspectives* (1999). Edited by Rene A. Carmona and Boris Rozovskii, American Math. Society, Providence, Rhode Island

[4] Pao-Liu Chow (2007). *Stochastic Partial Differential Equations*. Chapman & Hall/CRC, Boca Raton, Florida

[5] Gopinath Kallianpur and Jie Xiong (1995). *Stochastic Differential Equations in Infinite Dimensional Spaces*. Institute of Math. Statist. Lecture Notes—Monograph Series, Hayward, California

[6] Mehran Kardar (1987). Replica Bethe ansatz studies of two-dimensional interfaces with quenched random impurities. *Nuclear Phys.* B290 [FS20], 582–602

[7] Mehran Kardar, Giorgio Parisi, and Yi-Cheng Zhang (1986). Dynamic scaling of growing interfaces, *Phys. Rev. Lett.* **56***(9)*, 889–892

[8] Peter Kotelenez (2008). *Stochastic Ordinary and Stochastic Partial Differential Equations*, Springer, New York

[9] Nicolai V. Krylov (2006). On the foundations of the L_p-theory of stochastic partial differential equations. In: *Stochastic Partial Differential Equations and Applications—VII*, 179–191, Lecture Notes Pure Appl. Math., Chapman & Hall/CRC, Boca Raton, Florida

[10] Ernesto Medina, Terrence Hwa, and Mehran Kardar (1989). Burgers equation with correlated noise: Renormalization-group analysis and applications to directed polymers and interface growth. *Phys. Rev. A*, **38***(6)*, 3053–3075

[11] Pierre-Louis Lions and Panagiotis Souganidis (2000). Fully nonlinear stochastic pde with semilinear stochastic dependence. *C.R. Acad. Sci. Paris Sér. I Math.* **331***(8)*, 617–624

[12] S. Peszat and J. Zabczyk (2007). *Stochastic Partial Differential Equations with Lévy Noise*. Encyclopedia of Mathematics and Its Applications, Cambridge University Press, Cambridge

[13] Guisseppe Da Prato and Jerzy Zabczyk (1992). *Stochastic Equations in Infinite Dimensions*, Encyclopedia of Mathematics and Its Applications, Cambridge University Press, Cambridge

[14] Claudia Prévôt and Michael Röckner (2007). *A Concise Course on Stochastic Partial Differential Equations*. Lecture Notes in Mathematics **1905**, Springer-Verlag, Berlin, Heidelberg

[15] B. L. Rozovskiĭ (1990). *Stochastic Evolution Systems*. Kluwer Academic Publishing Group, Dordrecht (Translated from the original Russian by A. Yarkho, *Math. and Its Applicatons* (Soviet Series), 35)

[16] J. B. Walsh (1986). *An Introduction to Stochastic Partial Differential Equations*. In: Ecole d'Ete de Probabilites de Saint Flour XIV, Lecture Notes in Mathematics **1180**, 265–438

[17] Wojbor A. Woyczyński (1998). *Burgers–KPZ Turbulence*, Lecture Notes in Mathematics **1700**, Springer, Berlin

[18] Ya. B. Zeldovich, S. A. Molchanov, A. S. Ruzmaĭkin, and D. D. Solokov (1987). Intermittency in random media, *Uspekhi Fiz. Nauk* **152***(1)*, 3–32 (in Russian) [English translation in *Soviet Phys. Uspekhi* **30***(5)*, 353–369, 1987]

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