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Giuseppe Buttazzo · Aldo Pratelli
Sergio Solimini · Eugene Stepanov

Optimal Urban Networks via Mass Transportation

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Giuseppe Buttazzo
Dipartimento di Matematica
Università di Pisa
Largo Bruno Pontecorvo 5
56127 Pisa
Italy
buttazzo@dm.unipi.it

Aldo Pratelli
Dipartimento di Matematica
Università di Pavia
Via Ferrata 1,
27100 Pavia
Italy
aldo.pratelli@unipv.it

Sergio Solimini
Dipartimento di Matematica
Politecnico di Bari
Via Amendola 126/b
70126 Bari
Italy
solimini@dm.uniba.it

Eugene Stepanov
St. Petersburg University
of Information Technology, Mechanics
and Optics
Kronverkskij pr. 49
197101 St. Petersburg
Russia
stepanov.eugene@gmail.com

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Preface

The monograph is dedicated to a class of models of optimization of transportation networks (urban traffic networks or networks of railroads and highways) in the given geographic area. One assumes that the data on distributions of population and of services/workplaces (i.e. sources and sinks of the network) as well as the costs of movement with and without the help of the network to be constructed, are known. Further, the models take into consideration both the cost of everyday movement of the population and the cost of construction and maintenance of the network, the latter being determined by a given function on the total length of the network. The above data suffice, if one considers optimization in long-term prospective, while for the short-term optimization one also needs to know the transport plan of everyday movements of the population (i.e. the information on "who goes where"). Similar models can also be adapted for the optimization of networks of different nature, like telecommunication, pipeline or drainage networks. In the monograph we study the most general problem settings, namely, when neither the shape nor even the topology of the network to be constructed is a priori known.

To be more precise, given a region $\Omega \subseteq \mathbb{R}^N$, we will model the transportation network to be constructed by an a priori generic Borel set $\Sigma \subseteq \Omega$. We consider then the mass transportation problem in which the paths inside and outside the network Σ are charged differently. The aim is to find the best location for Σ, in order to minimize a suitable cost functional $\mathfrak{F}(\Sigma)$, which is given by the sum of the cost of transportation of the population, and the penalization term depending on the length of the network, which represents the cost of construction and maintenance of the network. To study the problem of existence of optimal solutions, we present first a relaxed version of the optimization problem, where the network is represented by a Borel measure rather than a set, and we prove the existence of a relaxed solution. We will study then the properties of optimal relaxed solutions (measures) and prove that, under suitable assumptions, the relaxed solution solve the original problem, i.e. in fact they correspond to rectifiable sets, and therefore can be called

classical solutions. However, it will be shown that in general the problem studied may have no classical solutions. We will also study some topological properties of optimal networks, like closedness and the number of connected components. In particular, we find rather sharp conditions on problem data, which ensure the existence of closed optimal networks and/or optimal networks having at most countably many connected components. Finally, we will prove a general regularity result on optimal networks. Namely, we will show that an optimal network is covered by a finite number of Lipschitz curves of uniformly bounded length, although it may have even uncountably many connected components.

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