Part I
Joseph Lipman: Notes on Derived Functors and Grothendieck Duality

Abstract

This is a polished version of notes begun in the late 1980s, largely available from my home page since then, meant to be accessible to mid-level graduate students. The first three chapters treat the basics of derived categories and functors, and of the rich formalism, over ringed spaces, of the derived functors, for unbounded complexes, of the sheaf functors $\otimes$, $\mathcal{H}om$, f_* and f^* (where f is a ringed-space map). Included are some enhancements, for concentrated (= quasi-compact and quasi-separated) schemes, of classical results such as the projection and Künneth isomorphisms. The fourth chapter presents the abstract foundations of Grothendieck Duality—existence and tor-independent base change for the right adjoint of the derived functor $\mathbf{R}f_*$ when f is a quasi-proper map of concentrated schemes, the twisted inverse image pseudofunctor for separated finite-type maps of noetherian schemes, some refinements for maps of finite tor-dimension, and a brief discussion of dualizing complexes.