Lecture Notes in Mathematics 1951

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A Nonlinear Transfer Technique for Renorming

Springer

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ISBN: 978-3-540-85030-4 e-ISBN: 978-3-540-85031-1
DOI: 10.1007/978-3-540-85031-1

Lecture Notes in Mathematics ISSN print edition: 0075-8434
ISSN electronic edition: 1617-9692

Library of Congress Control Number: 2008932180

Mathematics Subject Classification (2000): 46B03, 46B20, 46B26, 46T20, 54D20, 54E18, 54E25, 54E35

Cover design: SPi Publishing Services

Printed on acid-free paper

9 8 7 6 5 4 3 2 1

springer.com

Preface

Banach spaces are objects with a linear structure so linear maps have been considered the natural tool for transferring *good* norms from one Banach space to another. It is well known that a Banach space X admits an equivalent strictly convex (rotund) norm if there is a bounded linear one-to-one operator $T : X \to Y$ where Y has such a norm. For example, J. Lindenstrauss proved that in any reflexive space X there is such an operator $T : X \to c_0(\Gamma)$ for some set Γ. F. Dashiell and J. Lindenstrauss gave an example of a strictly convex renormable space without such an operator into $c_0(\Gamma)$ for any Γ. For that reason we are searching for a non linear transfer technique. We consider here locally uniformly rotund (**LUR**) norms, a property adding to strict convexity the coincidence of the weak and the norm topologies on the unit sphere. For these norms a class of non linear maps was not only more powerful but even more natural for this purpose, as evinced by the solution of an old open problem due to Kadec using this class of non linear maps. The scope of this technique is not restricted to that particular case but, on the contrary, offers a unified method of obtaining this renorming, roughly speaking, in all cases in which this is known to be possible.

We have been lecturing on these new techniques throughout the courses given in the Spring School of Paseky nad Jizerou in 1998; in the Workshop in Banach spaces, Prague 2000; and in the 28th, 30th and 31st Winter Schools of Lhota nad Rohanovem on Abstract Analysis, in 2000, 2002 and 2003, places where these notes had their genesis. We would like to thank Professors J. Lukes, J. Kottas, V. Zizler, P. Holický, L. Zajíček, J. Tiser, M. Fabian and O. Kalenda for their invitations and their warm hospitality. Different parts of these notes have also been presented in seminars and conferences, such as the Choquet, Godefroy, Rogalski, Saint Raymond Analysis Seminar, University Pierre and Marie Curie, Paris VI, 1999 and 2001; Laboratoire de mathématiques pures de Bordeaux, University of Bordeaux, 1999; Functional Analysis Seminar and Analytic Topology Seminar, Mathematical Institute, Oxford University 2001 and 2002; VII Conference on Function Theory on

Infinite Dimensional Spaces, UCM, Madrid in 2001; Geometry of Banach spaces, Mathematisches Forschungsinstitut Oberwolfach, Germany, 2003; Interplay between Topology and Analysis at the International Congress Massee, Borovets, Bulgaria, 2003; Spring School on Non Separable Banach Spaces, Paseky nad Jizerou in 2004, and the Contemporary Ramifications of Banach space theory conference in honour of Joram Lindenstrauss and Lior Tzafriri, Institute of Advance Studies, Hebrew University of Jerusalem, 2005. We would like to thank G. Godefroy, R. Deville, C. J. K. Batty, P. Collins, J. L. González Llavona, D. Azagra, M. Jiménez, H. König, J. Lindenstrauss, N. Tomczak-Jaegermann, P. Kenderov, J. Lukes, M. Fabian, P. Hájek, V. Zizler, L. Tzafriri, T. Szankowski and M. Zippin for their excellent qualities as hosts and their grace and patience as audiences. J. Lindenstrauss deserves special gratitude for his insightful comments and encouragement with the topics presented here. Thanks are also due to I. Namioka for reading these notes, providing us with different points of view and excellent mathematical ideas. We wish to thank R. Haydon for many helpful suggestions and for our always interesting and stimulating conversations. Last, but certainly not least, we would like to express our debt to G. Godefroy, who was the first mathematician to suggest to us the idea of publishing all this material together, constantly encouraging us to finish our project.

Therefore despite the fact that the content of these notes is new and has not been published elsewhere, they have a self-contained and unified approach to the study of the existence of local uniformly rotund norms with a new point of view. As a result we hope they are accessible for readers with a basic knowledge of Functional Analysis and Set Theoretic Topology.

We study maps from a normed space X to a metric space Y which provide a **LUR** renorming in X. These maps are just those which satisfy two conditions that we call σ-slicely continuity and co-σ-continuity. Our main goal here is to characterize both properties, applying them as a new frame for **LUR** renormings. The characterization is an interplay between Functional Analysis, Optimization and Topology. We use ε-subdifferentials of Lipschitz functions and apply methods of metrization theory to the study of weak topologies. For example we find that any one-to-one operator T from X (reflexive, or even weakly countably determined) into $c_0(\Gamma)$ satisfies both conditions. Nevertheless our maps can be far away from the class of linear maps even when Y is a normed space. For instance the duality map from X into its dual is σ-slicely continuous if the norm of X is Fréchet differentiable. If in addition the dual norm is Gâteaux differentiable, then the duality map is co-σ-continuous and X is **LUR** renormable.

Murcia and Valencia,
July 2007

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Acknowledgement

The authors would like to thank all who supported the research for this work.
Aníbal Moltó has been partially supported by BFM2003–07540/MATE and MTM2007-064521 del Ministerio de Ciencia y Tecnología MCIT y FEDER.
José Orihuela has been partially supported by PI–55/00872/FS/01, and 00690/PI/2004 Fundación Séneca CARM and BFM 2002–01719, MTM2005-08379 del Ministerio de Ciencia y Tecnología MCIT and Ministerio de Educación y Ciencia.
Stanimir Troyanski has been partially supported by Grant MM–1401/04 of the NSFR of Bulgaria, by PI–55/00872/FS/01, and 00690/PI/2004 Fundación Séneca CARM, by BFM 2002–01719, MTM2005-08379 del Ministerio de Ciencia y Tecnología MCIT and Ministerio de Educación y Ciencia and by a research project of the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences.
Manuel Valdivia has been partially supported by MEC and FEDER Project MTM2005-08210.

Contents

List of Symbols