Lecture Notes in Mathematics 1945

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Mathematical Epidemiology

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Springer

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ISBN: 978-3-540-78910-9 e-ISBN: 978-3-540-78911-6
DOI: 10.1007/978-3-540-78911-6

Lecture Notes in Mathematics ISSN print edition: 0075-8434
ISSN electronic edition: 1617-9692

Library of Congress Control Number: 2008923889

Mathematics Subject Classification (2000): 92D30, 92D25, 34D05, 60J80, 60G05

Cover design: WMXDesign GmbH, Heidelberg
The artwork on the cover page was designed by Ping Yan

Printed on acid-free paper

9 8 7 6 5 4 3 2 1

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Preface

Mathematical epidemiology has a long history, going back to the smallpox model of Daniel Bernoulli in 1760. Much of the basic theory was developed between 1900 and 1935, and there has been steady progress since that time. More recently, models to evaluate the effect of control measures have been used to assist in the formulation of policy decisions, notably for the foot and mouth disease outbreak in Great Britain in 2001. The SARS (Severe Acute Respiratory Syndrome) epidemic of 2002–2003 aroused great interest in the use of mathematical models to predict the course of an infectious disease and to compare the effects of different control strategies. This revived interest has been reinforced by the current threat of an influenza pandemic.

Mathematical epidemiology differs from most sciences as it does not lend itself to experimental validation of models. Experiments are usually impossible and would probably be unethical. This gives great importance to mathematical models as a possible tool for the comparison of strategies to plan for an anticipated epidemic or pandemic, and to deal with a disease outbreak in real time.

In response to the SARS epidemic, a team was formed by a Canadian center, MITACS (Mathematics for Information Technology and Complex Systems) to work on models for the transmission dynamics of infectious diseases, with a specific goal of evaluating possible management strategies. This team soon recognized that for mathematical modeling to be of assistance in making health policy decisions, it would be necessary to increase the number of mathematical modelers in epidemiology and also to persuade decision makers in the health sciences that mathematical modeling could be useful for them. In pursuit of these goals, a summer school in mathematical epidemiology was developed in 2004 for graduate students from mathematical and biological sciences. This school consisted of a series of lectures on various topics in mathematical epidemiology together with projects done by groups of students, each group containing students from various disciplines and with different levels of experience. In the summer of 2006, another summer school was held, again for a mixed group but this time including a substantial number

of people from epidemiology and health sciences. The 2006 summer school also included a few public lectures that covered a wide range of issues and diseases of great interest to public health, illustrating the general framework and abstract mathematical theory in an applied setting. The experience from these courses was that the projects were an essential valuable component of the school, and the mixing of students in the project groups had a very positive effect for communication between disciplines, but also that differences in mathematical backgrounds caused some difficulties.

This book consists of lecture notes intended for such a school. They cover the main aspects of mathematical modeling in epidemiology and contain more than enough material for a concentrated course, giving students additional resource material to pursue the subject further. Our goal is to persuade epidemiologists and public health workers that mathematical modeling can be of use to them. Ideally, it would teach the art of mathematical modeling, but we believe that this art is difficult to teach and is better learnt by doing. For this reason, we have settled on the more modest goal of presenting the main mathematical tools that will be useful in analyzing models and some case studies as examples. We hope that understanding of the case studies will give some insights into the process of mathematical modeling.

We could give a flow chart for the use of this volume but it would not be very interesting, as there is very little interdependence between chapters. Everyone should read the opening chapter as a gentle introduction to the subject, and Chap. 2 on compartmental models is essential for all that follows. Otherwise, the only real chapter dependence is that Chap. 10 requires an understanding of Chap. 3. The first four chapters are basic material, the next six chapters are developments of the basic theory, and the last four chapters are case studies on childhood diseases, influenza, and West Nile virus. The two case studies on influenza deal with different aspects of the disease and do not depend on each other. There are also some suggested projects, taken in part from the recent book "A Course in Mathematical Biology: Quantitative Modeling with Mathematical and Computational Methods" by Gerda de Vries, Thomas Hillen, Mark Lewis, Johannes Müller, and Birgit Schönfisch, *Mathematical Modeling and Computation* 12, SIAM, Philadelphia (2006).

The necessary mathematical background varies from chapter to chapter but a knowledge of basic calculus, ordinary differential equations, and some matrix algebra is essential for understanding this volume. In addition, Chaps. 3, 4, and 10 require some background in probability. Review notes on calculus, matrix algebra, differential equations, and probability have been prepared and may be downloaded at the web site of the Center for Disease Modelling (www.cdm.yorku.ca). Some chapters use more advanced mathematical topics. Some topics in linear algebra beyond elementary matrix theory are needed for Chaps. 6–8. Hopf bifurcations are used in Chaps. 5 and 13. Some knowledge of partial differential equations is needed for Chapters 8, 9, and 13. Preparation of review notes on these topics is in process.

Students from epidemiology and health sciences would probably need some review of basic mathematics, and a course for students with a meagre mathematical background should probably be restricted to the first two chapters and the case studies in Chaps. 12 and 14. A course for students with a strong mathematical background could include any of the chapters depending on the interests of instructors and students. For example, a course emphasizing stochastic ideas could consist of the first four chapters, Chap. 10, plus some of the case studies in the last four chapters. We believe that every course should include some case studies.

We plan to use this volume as text material for future courses of various lengths and with a variety of audiences. One of the main goals of the courses on which this volume is based was to include students from different disciplines. Our experience suggests that a future course aimed at a mixed group should include a variety of non-mathematical case study descriptions and should probably begin with separate tracks for "calculus users" who are comfortable with basic mathematics but have little or no experience with epidemiology, and "calculus victims" who may have studied calculus but in a form that did not persuade them of the value of applications of mathematics to other sciences. The negative experiences that many students in the health sciences may have had in the past are a substantial obstacle that needs to be overcome.

The chapters of this book are independent units and have different levels of difficulty, although there is some overlap. Tremendous efforts were made to ensure that these lectures are coherent and complementary, but no attempt has been made to achieve a unified writing style, or even a unified notation for this book. Because mathematical epidemiology is a rapidly developing field, one goal of any course should be to encourage students to go to the current literature, and experience with different perspectives should be very helpful in being able to assimilate current developments.

These lecture notes would have been impossible without the two summer schools funded by MITACS, as well as the Banff International Research Station for Mathematical Innovation and Discovery (BIRS), The Fields Institute for Research in Mathematical Sciences, the Mathematical Sciences Research Institute and the Pacific Institute for the Mathematical Sciences. We thank these funding agencies for their support, as well as BIRS and York University for supplying physical facilities. We thank Dr. Guojun Gan and Dr. Hongbin Guo for help in assembling the book manuscript. Finally, we thank all the lecturers as well as Dr. Julien Arino and Dr. Lin Wang for technical support during the summer schools, and the summer school students who contributed much useful feedback.

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February 2008

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